Lezioni dottorali, Università degli Studi di Napoli "Parthenope"

Napoli, 21–27 October 2016

Nonlinearity, Tipping Points & Chaos in the Climate Sciences

Michael Ghil

**Ecole Normale Supérieure, Paris, and
University of California, Los Angeles**

Please visit these sites for more info.

https://dept.atmos.ucla.edu/tcd
http://www.environnement.ens.fr/

Overall Outline

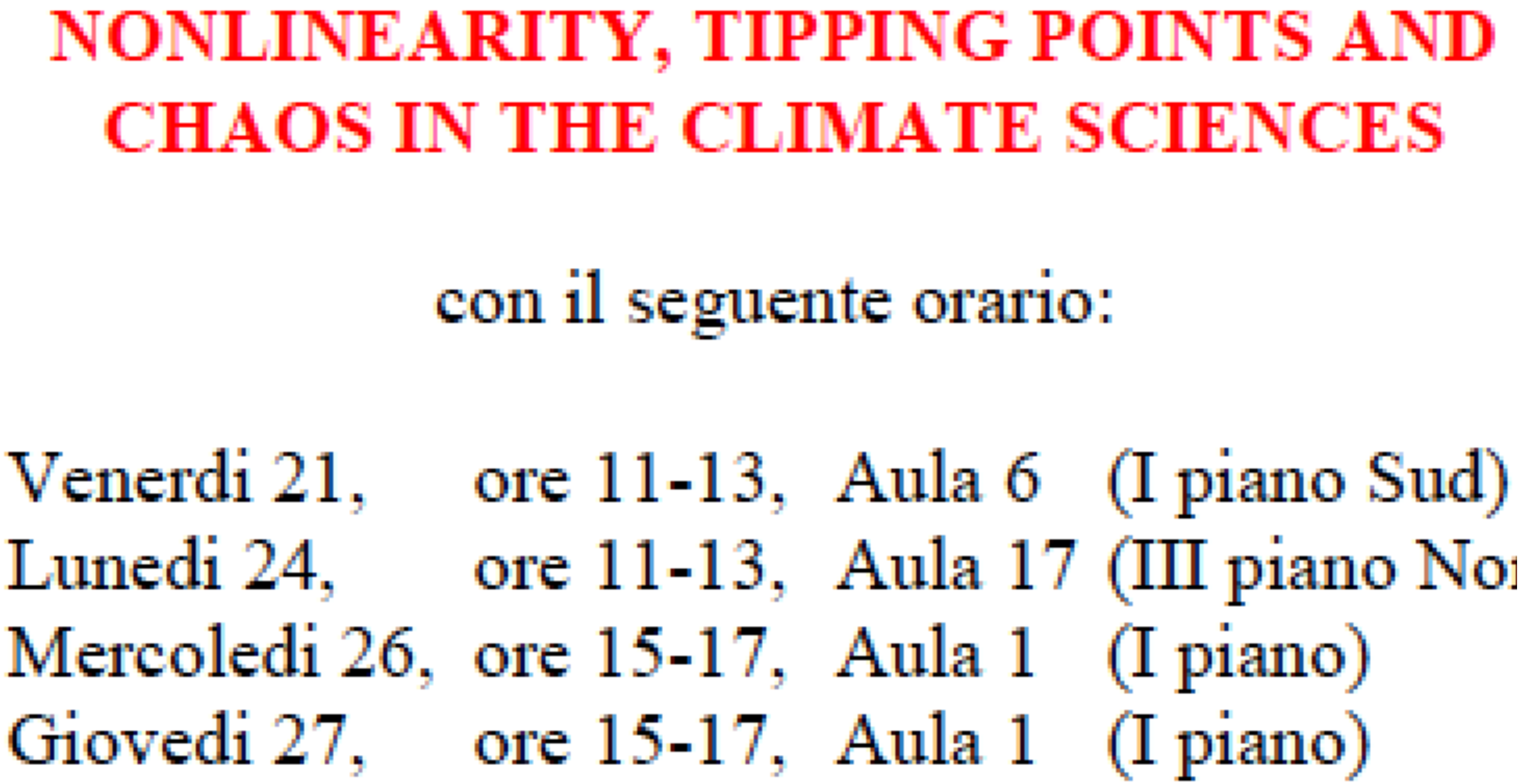
- **Lecture I: EBMs⁽⁺⁾, paleoclimate & “tipping points”**

➔**Lecture II: The wind-driven ocean circulation**

- **Lecture III: Advanced spectral methods–SSA⁽±⁾ *et al*.**
- **Lecture IV: Nonlinear & stochastic models—RDS⁽❖⁾**

(+) EBM = Energy balance model
(±) SSA = Singular-spectrum analysis
(❖) RDS = Random dynamical system

Dottorato di ricerca
“Fenomeni e rischi ambientali”
Serie di quattro lezioni dal Professore M. Ghil



NONLINEARITY, TIPPING POINTS AND CHAOS IN THE CLIMATE SCIENCES

con il seguente orario:

Venerdi 21,	ore 11-13,	Aula 6	(I piano Sud)
Lunedi 24,	ore 11-13,	Aula 17	(III piano No
Mercoledi 26,	ore 15-17,	Aula 1	(I piano)
Giovedi 27,	ore 15-17,	Aula 1	(I piano)

Motivation

- The ***North Atlantic Oscillation (NAO)*** is a leading mode of variability of the Northern Hemisphere and beyond.
- It affects ***the atmosphere and oceans*** on several time and space scales.
- Its ***predictive understanding*** could help interannual and decadal-scale climate prediction over and around the North Atlantic basin.
- The ***hierarchical modeling*** approach allows one to give proper weight to the understanding provided by the models *vs*. their realism, respectively.
- Back-and-forth between ***"toy"*** (conceptual) and ***detailed*** ("realistic") ***models***, and between ***models*** and ***data***.

Joint work with *S. Brachet* (SHOM), *F. Codron* (LMD), *H.A. Dijkstra* (Utrecht U.), *Y. Feliks* (IIBR), *S. Jiang*, *F.-F. Jin* (U. Hawaii), *H. Le Treut* (IPSL), *A.W. Robertson* (Columbia U.), *E. Simonnet* (INLN) & *S. Speich* (ENS)

Climate and Its Sensitivity

Let's say CO_2 doubles:

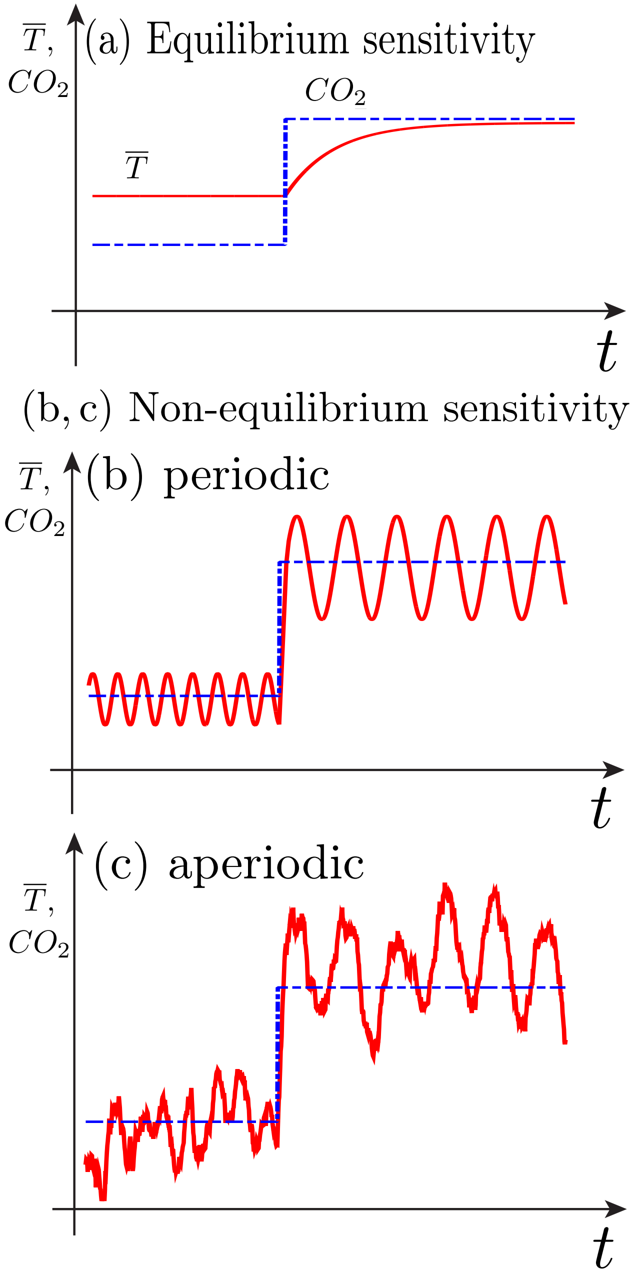
How will "climate" change?

1. Climate is in stable equilibrium (fixed point); if so, mean temperature will just shift gradually to its new equilibrium value.

2. Climate is purely periodic; if so, mean temperature will (maybe) shift gradually to its new equilibrium value. But how will the period, amplitude and phase of the limit cycle change?

3. And how about some "real stuff" now: chaotic + random?

Ghil (in *Encycl. Global Environmental Change*, 2002)



Lezioni dottorali, Università degli Studi di Napoli "Parthenope"

Napoli, 24 October 2016

The Wind-Driven Ocean Circulation and Global Change

Michael Ghil

Ecole Normale Supérieure, Paris, and
University of California, Los Angeles

Joint work with many people over the years; most recently **M.D. Chekroun, A. Groth & D. Kondrashov** (UCLA) + **Y. Feliks** (IIBR) + **S. Pierini** (U. Napoli-Parthenope) + **L. De Cruz, J. Demaeyer** & **S. Vannitsem** (RMI, Brussels)

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- Introduction: the NAO and the oceans' wind-driven circulation
- The low-frequency variability
    - bifurcations in a toy model
        => multiple equilibria, periodic and chaotic solutions
    - some intermediate model results
- Atmospheric impacts
    - simple and intermediate models + GCMs
- Some data analysis – atmospheric and oceanic
- Some promising NAO results
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- Conclusions
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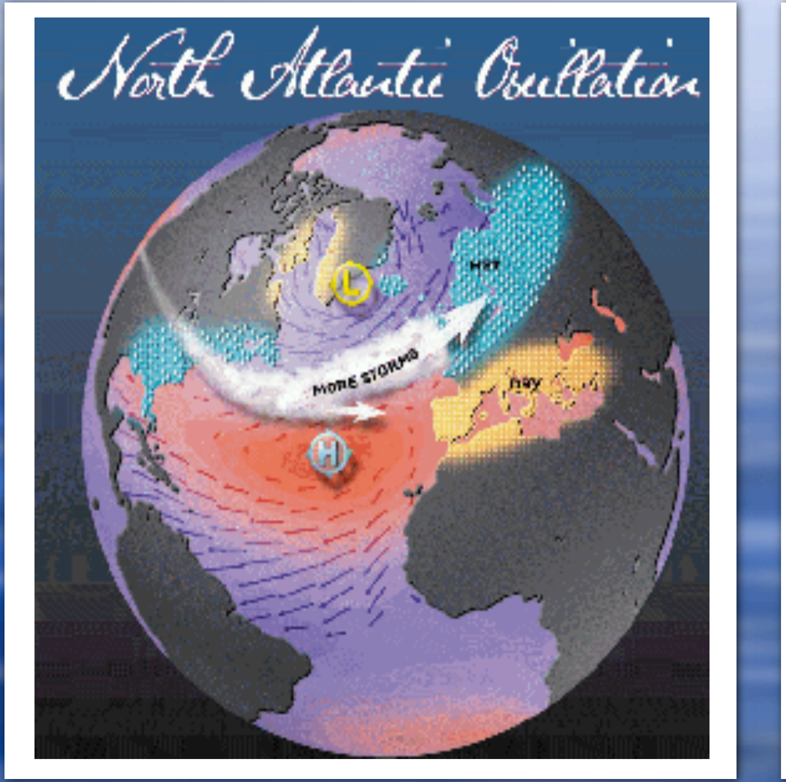
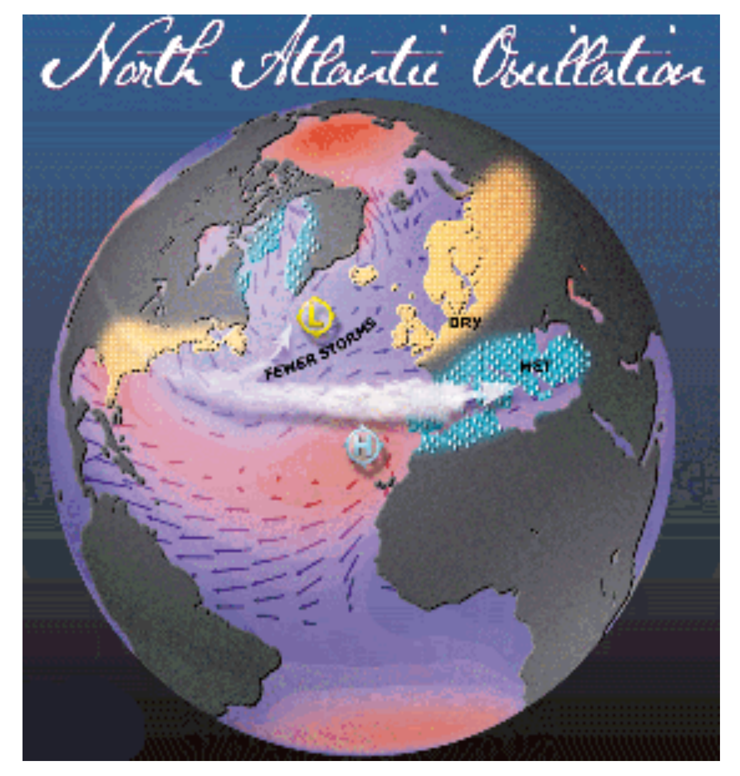
- Some references

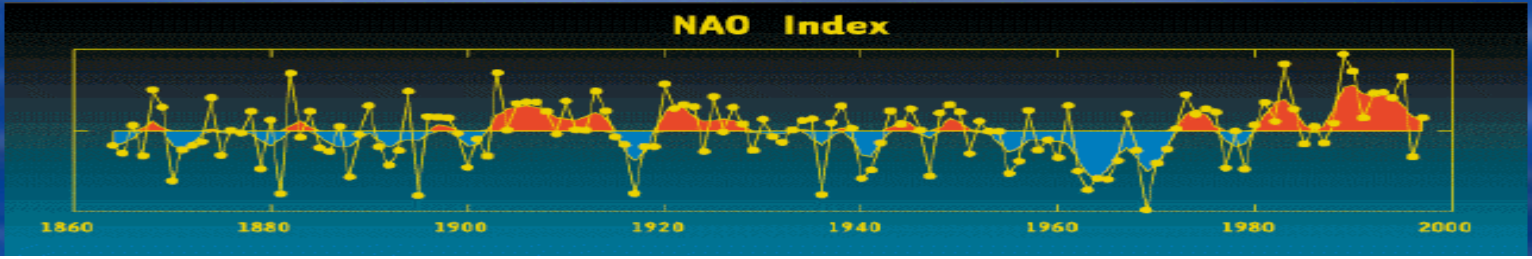
The North Atlantic Oscillation (NAO)

Positive phase

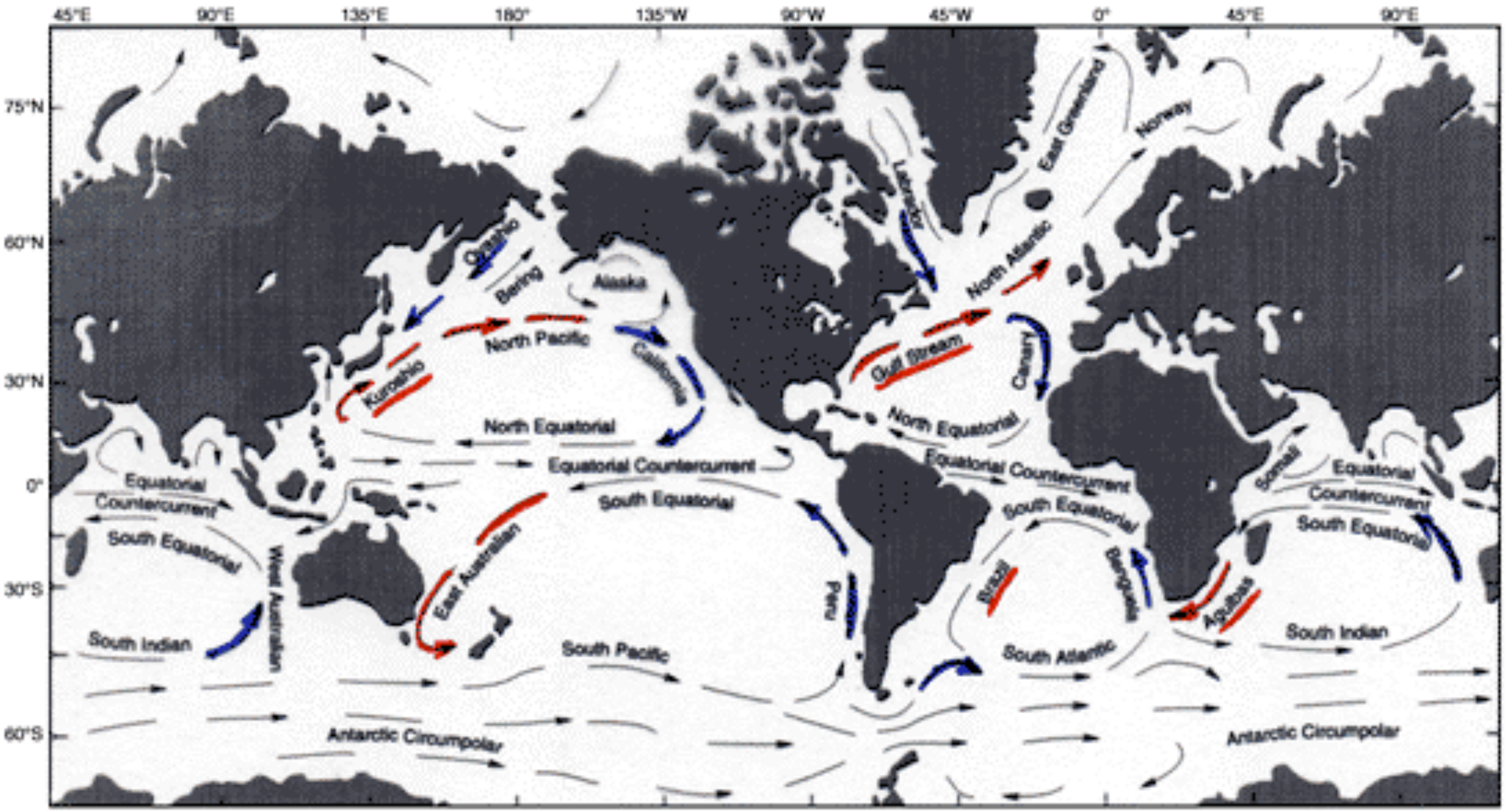


Negative phase





An example of bifurcations and hierarchical modeling: The oceans' wind-driven circulation



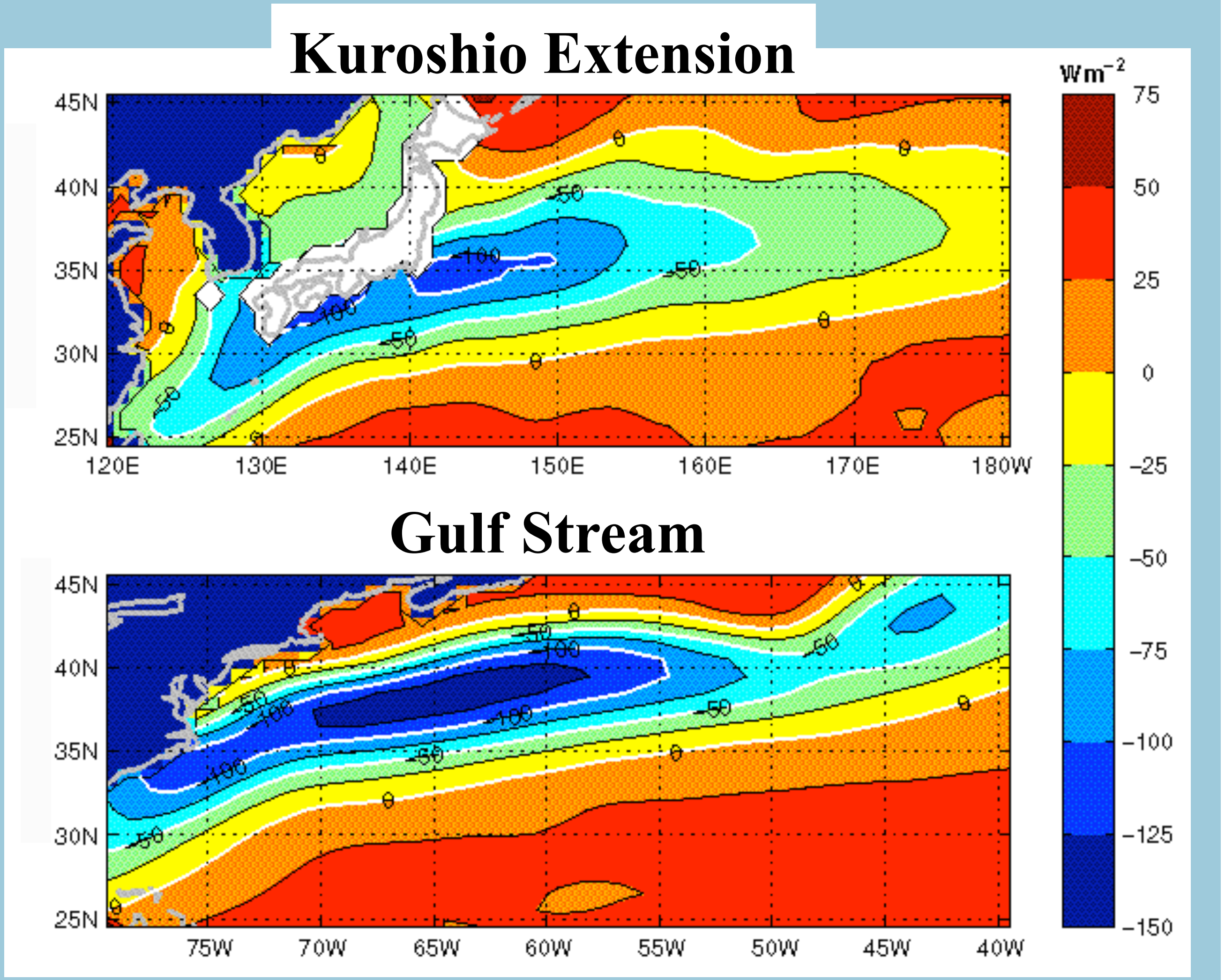
J. Apel (1987), Principles of Ocean Physics

The mean surface currents are (largely) wind-driven

Annual Mean Net Surface Heat Flux

Large heat loss balanced by poleward heat transport (advection)

Latent heat flux is large relative to sensible.



Kuroshio Extension

Gulf Stream

Kelly, Jan 2009

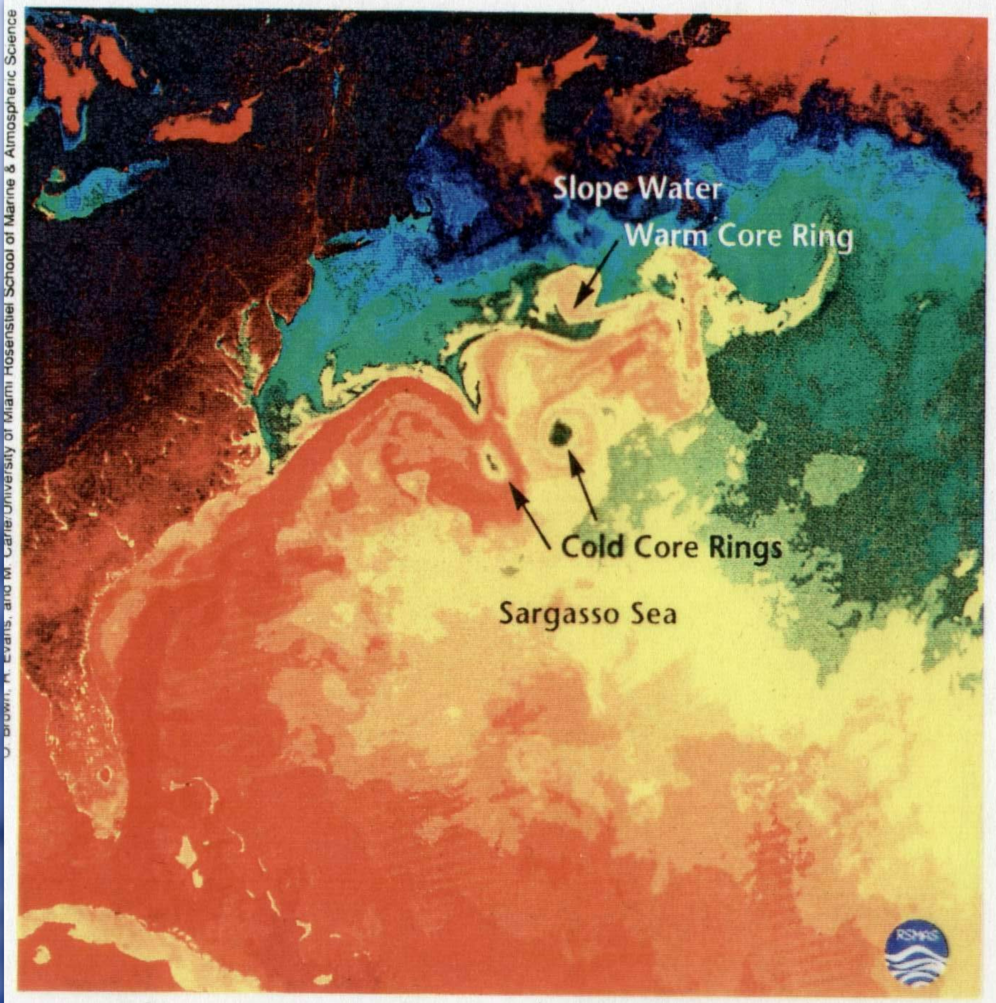
APPLIED PHYSICS LABORATORY
University of Washington

Southhampton Oceanography Centre

The gyres and the eddies

Many scales of motion, dominated in the mid-latitudes by (i) ***the double-gyre circulation***; and (ii) ***the rings and eddies***.

Much of the focus of physical oceanography over the ‘70s to ‘90s has been with the “***meso-scale***”: the meanders, rings & eddies, and the associated two-dimensional and quasi-geostrophic ***turbulence***.



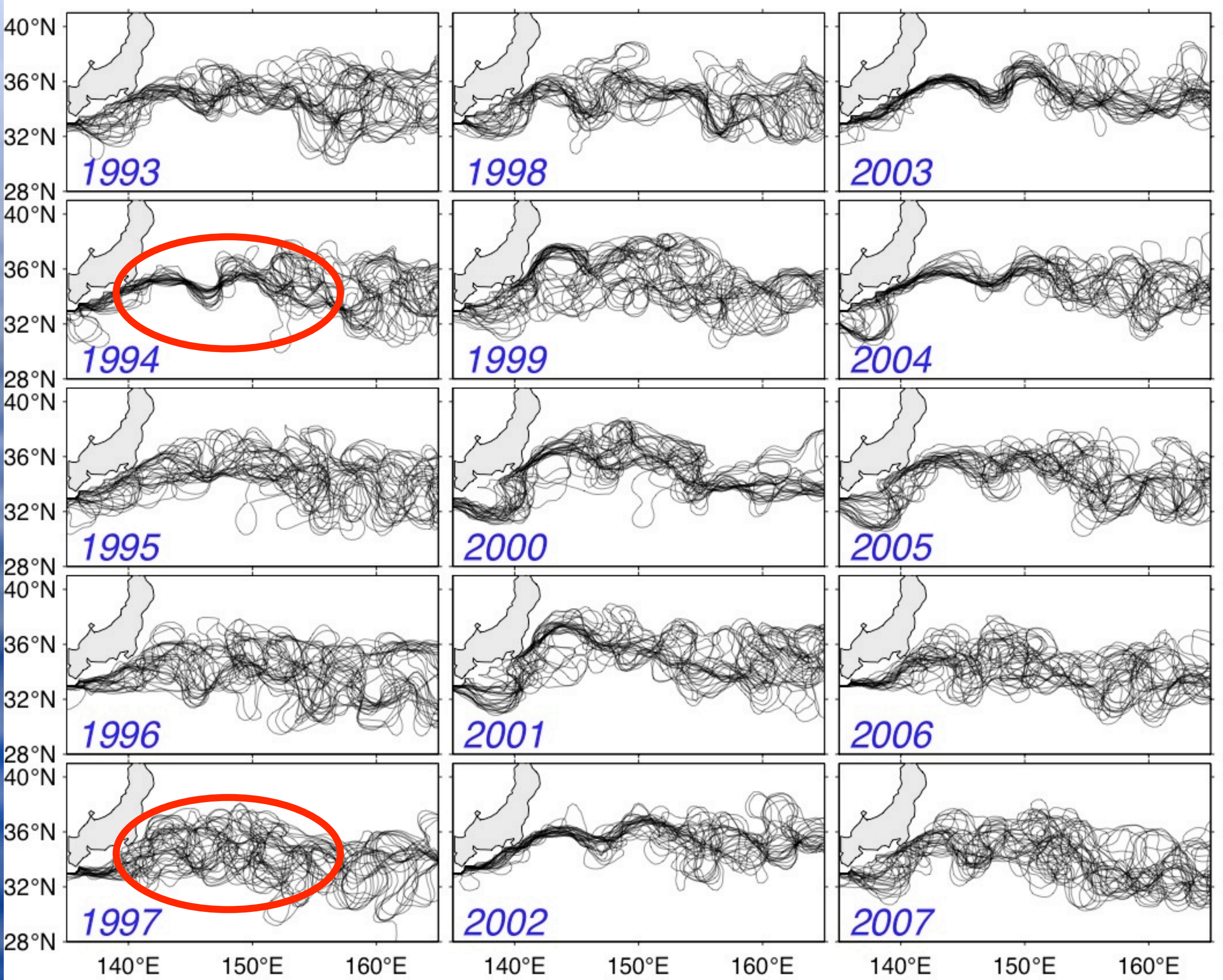
O. Brown, R. Evans, and M. Carle/University of Miami Rosenstiel School of Marine & Atmospheric Science

Based on SSTs, from satellite IR data

Kuroshio Extension (KE) Path Changes

Monthly paths from altimeter:

Stable vs. unstable periods



Qiu & Chen
(*Deep-Sea Res.*, 2009)

Modeling Hierarchy for the Oceans

Ocean models

- 0-D: box models – chemistry (BGC), paleo
- 1-D: vertical (mixed layer, thermocline)
- 2-D – meridional plane – buoyancy-driven (THC or MOC)
  → also 2.5-D: a little longitude dependence
  – horizontal – wind-driven
  → also 2.5-D: reduced-gravity models (n.5)
- 3-D: OGCMs – simplified
  – with bells & whistles (“kitchen sink”)

Coupled O-A models

- Idealized (0-D & 1-D): intermediate couple models (ICM)
- Hybrid (HCM) – diagnostic/statistical atmosphere
  – highly resolved ocean
- Coupled GCM (3-D): CGCM

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The double-gyre circulation and its low-frequency variability

An “intermediate” model of the mid-latitude, wind-driven ocean circulation: 20-km resolution, about 15 000 variables

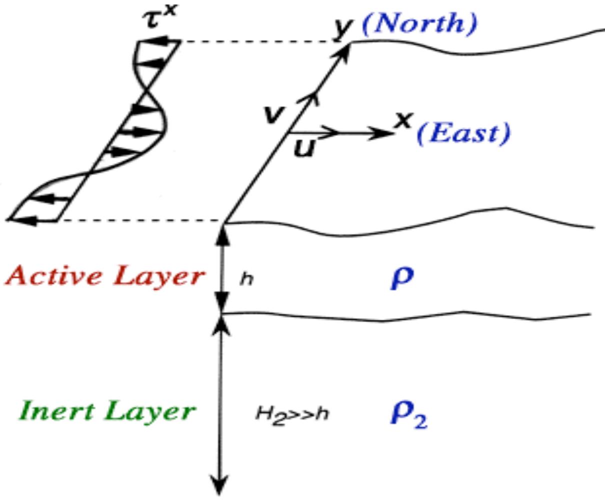
Shallow-water model

$$\begin{aligned}
\frac{\partial U}{\partial t} + \nabla\cdot(\mathbf{u}U) &= -g'h\frac{\partial h}{\partial x} + fV + \underline{\alpha_A} A\nabla^2 U - RU - \underline{\alpha_\tau}\frac{\tau^x}{\rho} \\
\frac{\partial V}{\partial t} + \nabla\cdot(\mathbf{u}V) &= -g'h\frac{\partial h}{\partial y} - fU + \underline{\alpha_A} A\nabla^2 V - RV \\
\frac{\partial h}{\partial t} &= -(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y})
\end{aligned}$$

where

$$U\hat{e_x} + V\hat{e_y} = h\mathbf{u} = h(u\hat{e_x} + v\hat{e_y})$$

g': reduced gravity $(= g(\rho_2 - \rho)/\rho)$
A: viscosity coefficient $(= 300\ \mathrm{m^2 s^{-1}})$
R: Rayleigh coefficient $(= 1/200\ \mathrm{day^{-1}})$
τ^x: wind stress $= \tau_o \cos 2\pi/L (\tau_o = 1\ \mathrm{dyn\ cm^{-2}}\ \&L = 2000\mathrm{km})$



Reduced gravity (1.5 - layer) model

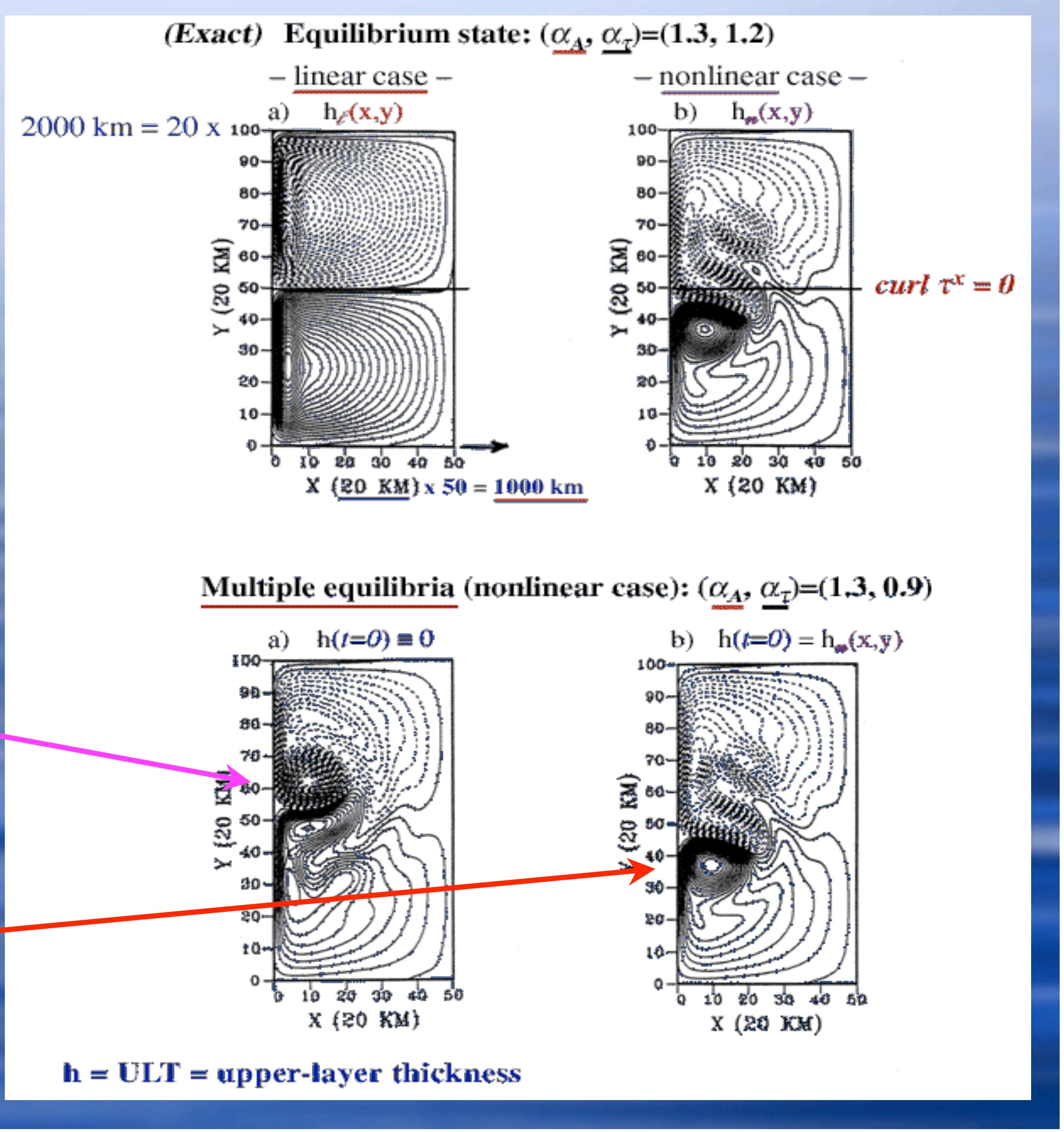
S. Jiang, F.-F. Jin & M. Ghil (1995) J. Phys. Oceanog., 25: 764-786

The JJG model's equilibria

Nonlinear (advection) effects break the (near) symmetry: (perturbed) pitchfork bifurcation?

Subpolar gyre dominates

Subtropical gyre dominates



(Exact) **Equilibrium state: (α_A, α_τ)=(1.3, 1.2)**

– linear case – a) $h_\ell(x,y)$

– nonlinear case – b) $h_\infty(x,y)$

Multiple equilibria (nonlinear case): (α_A, α_τ)=(1.3, 0.9)

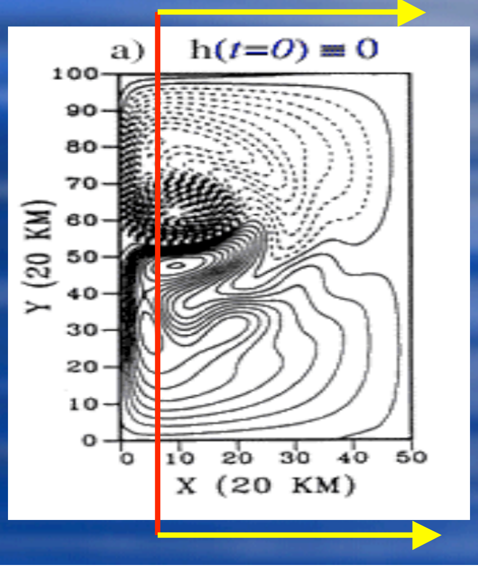
a) $h(t=0) \equiv 0$

b) $h(t=0) = h_\infty(x,y)$

h = ULT = upper-layer thickness

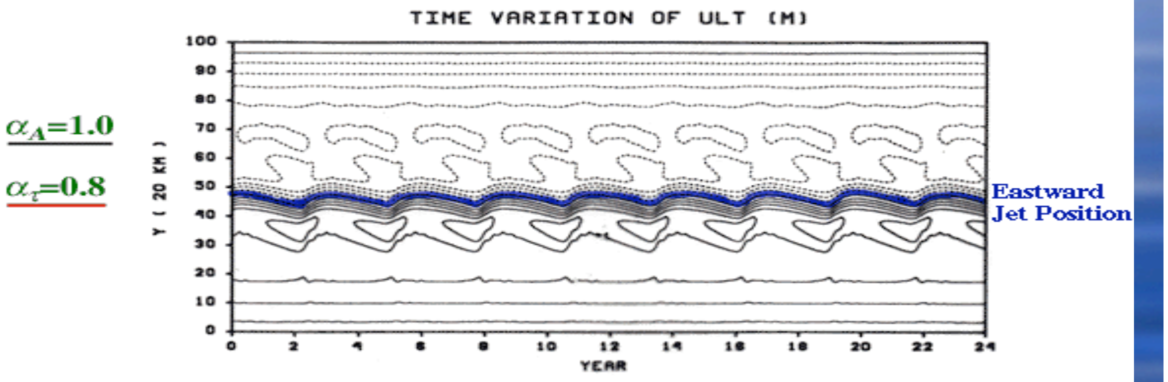
Time-dependent solutions: periodic and chaotic

To capture space-time dependence, meteorologists and oceanographers often use Hovmöller diagrams

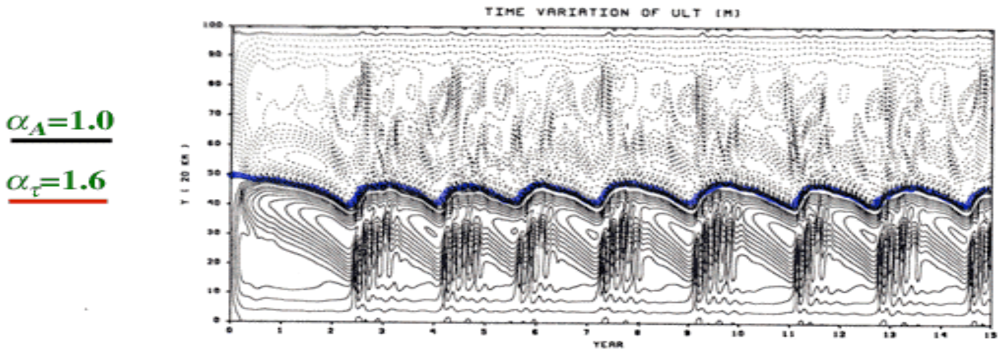


Time-dependent solutions

1. Periodic, w/ interannual period (2.8 years)



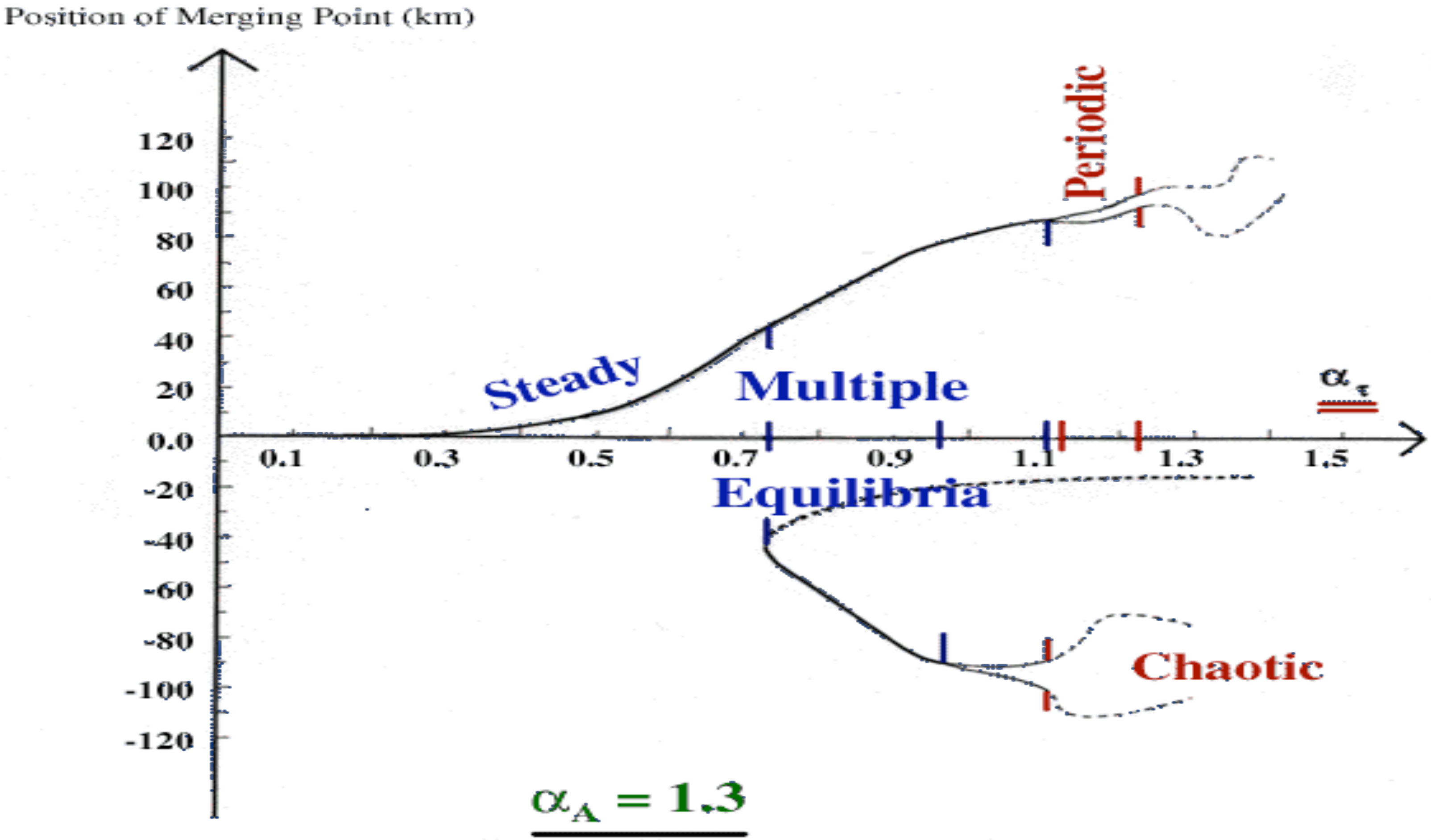
2. Aperiodic (weakly chaotic)



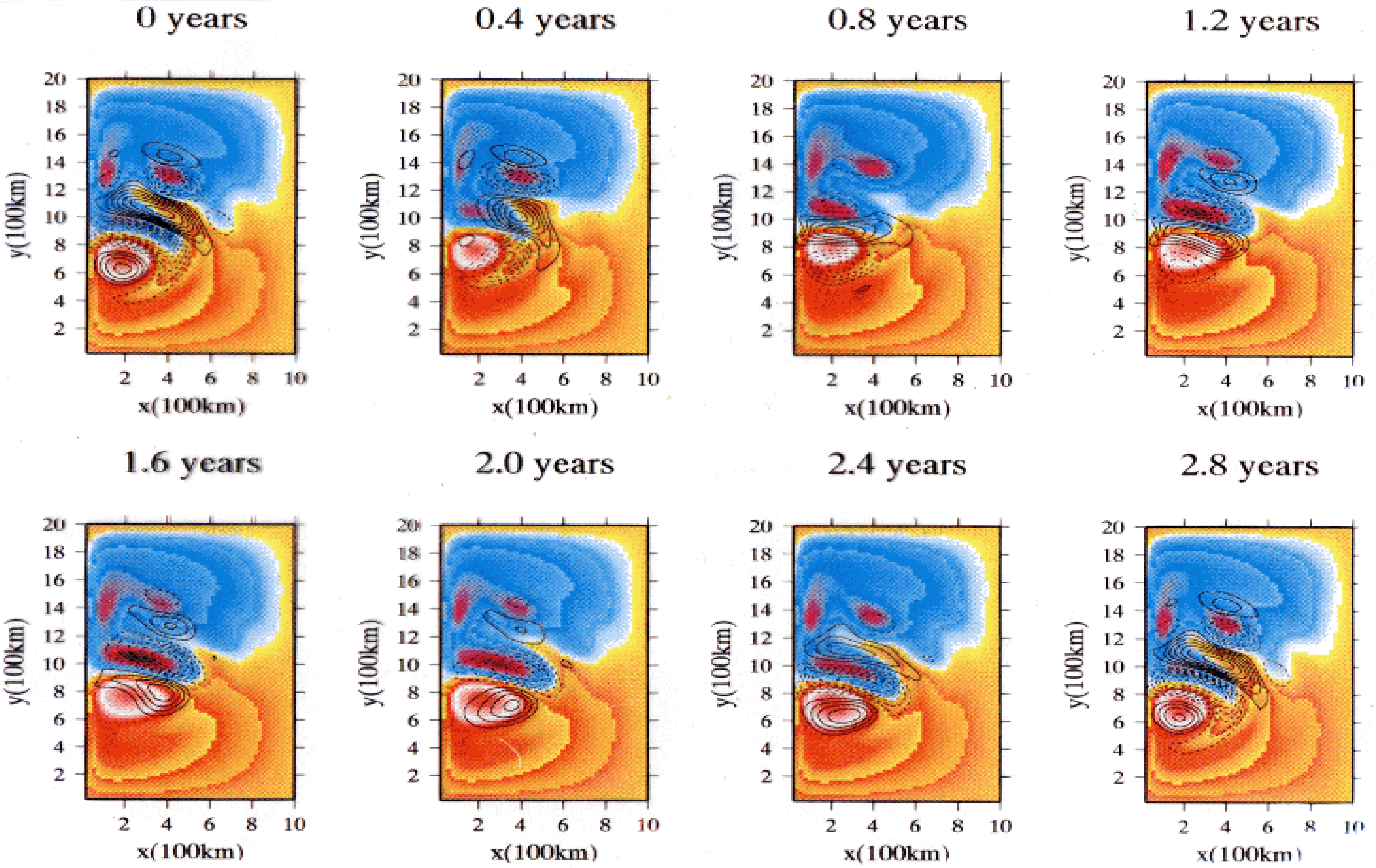
Poor man's continuation method

Bifurcation diagram

Perturbed pitchfork + Hopf + transition to chaos



Interannual variability: relaxation oscillation



0 years

0.4 years

0.8 years

1.2 years

1.6 years

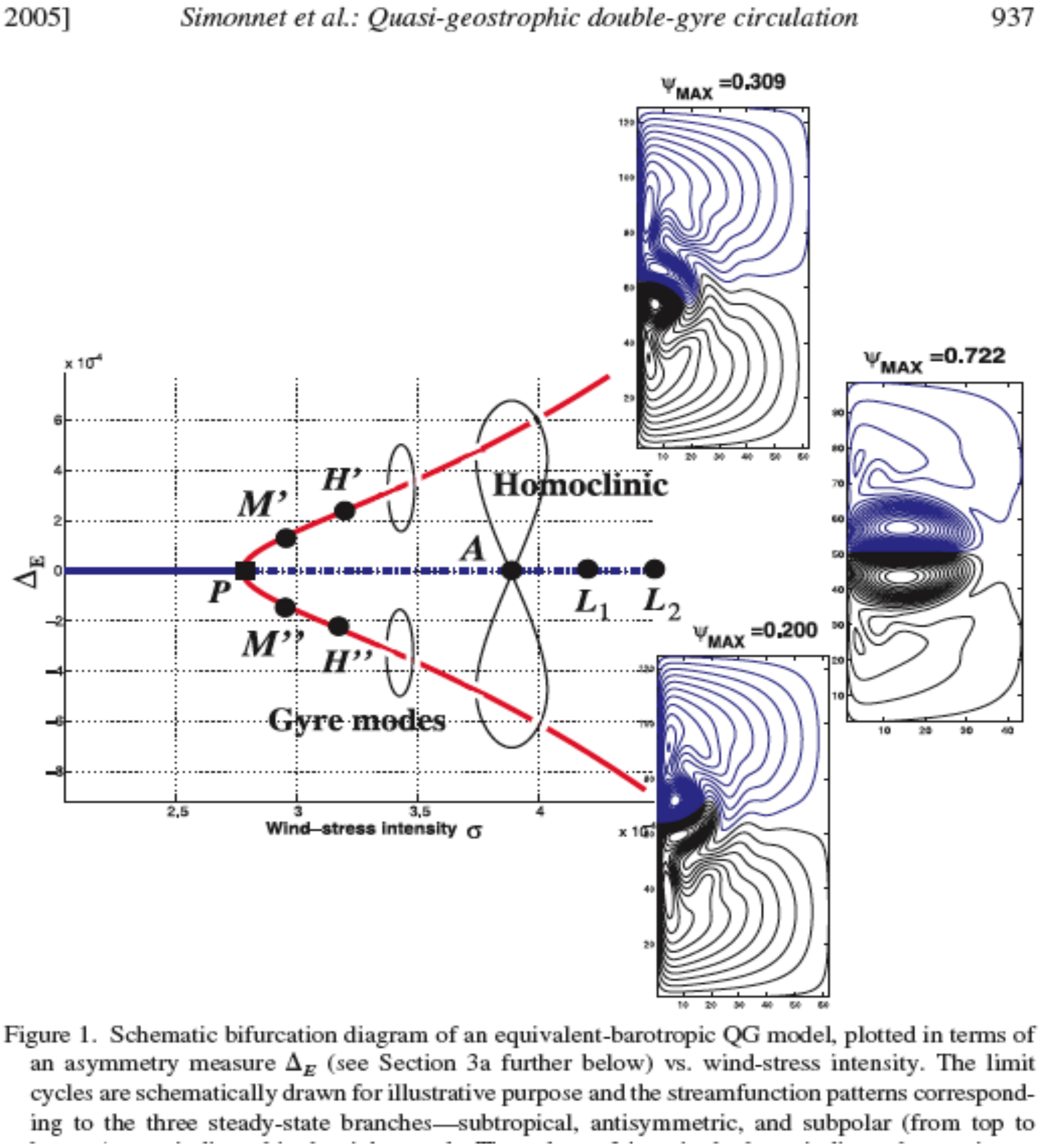
2.0 years

2.4 years

2.8 years

Global bifurcations in "intermediate" models

Bifurcation tree in a QG, equivalent-barotropic, high-resolution (10 km) model: pitchfork, mode-merging, Hopf, and homoclinic!

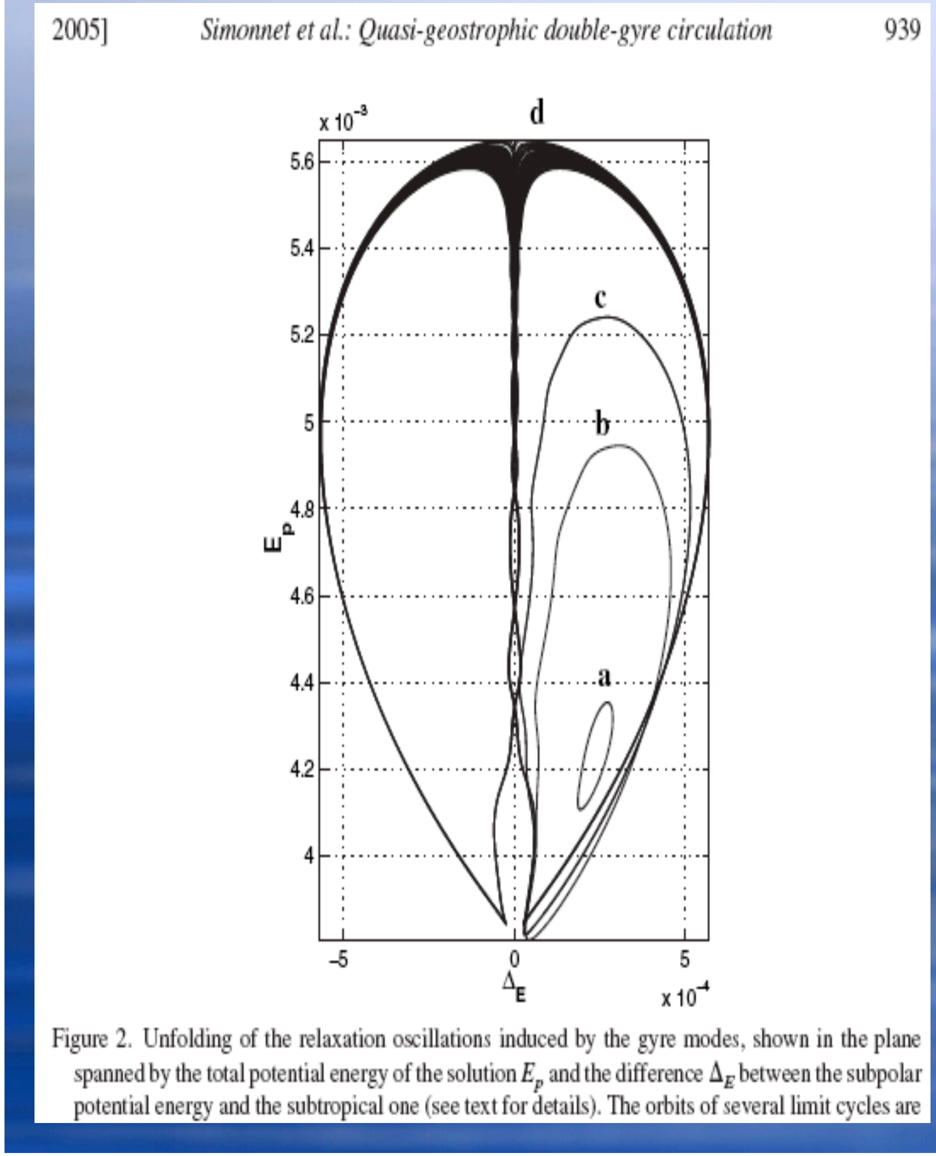


2005] *Simonnet et al.: Quasi-geostrophic double-gyre circulation* 937

Figure 1. Schematic bifurcation diagram of an equivalent-barotropic QG model, plotted in terms of an asymmetry measure Δ_E (see Section 3a further below) vs. wind-stress intensity. The limit cycles are schematically drawn for illustrative purpose and the streamfunction patterns corresponding to the three steady-state branches—subtropical, antisymmetric, and subpolar (from top to

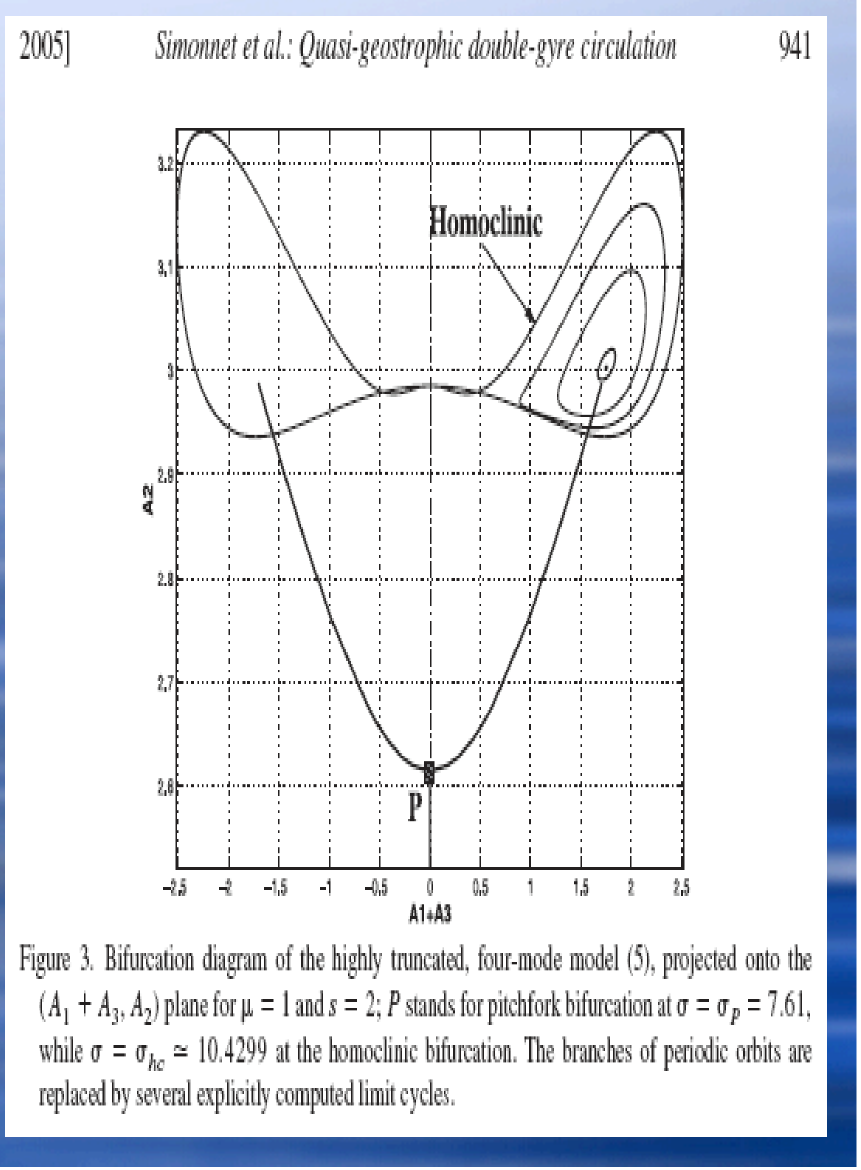
Simonnet, Ghil & Dijkstra
(*J. Mar. Res.*, 2005)

Homoclinic orbit: numerical and analytical



2005] *Simonnet et al.: Quasi-geostrophic double-gyre circulation* 939

Figure 2. Unfolding of the relaxation oscillations induced by the gyre modes, shown in the plane spanned by the total potential energy of the solution E_p and the difference Δ_E between the subpolar potential energy and the subtropical one (see text for details). The orbits of several limit cycles are



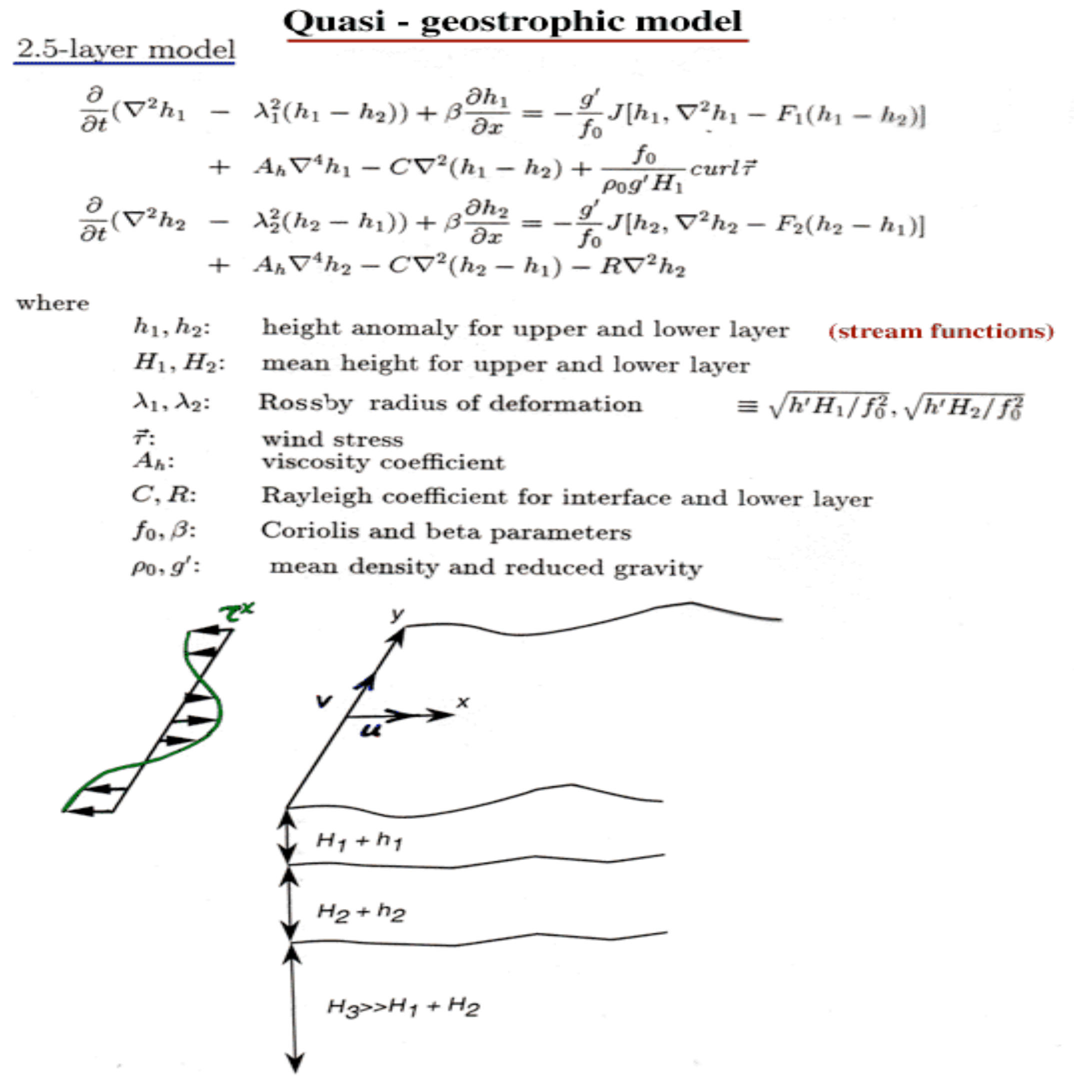
2005] *Simonnet et al.: Quasi-geostrophic double-gyre circulation* 941

Figure 3. Bifurcation diagram of the highly truncated, four-mode model (5), projected onto the $(A_1 + A_3, A_2)$ plane for $\mu = 1$ and $s = 2$; P stands for pitchfork bifurcation at $\sigma = \sigma_p = 7.61$, while $\sigma = \sigma_{hc} \simeq 10.4299$ at the homoclinic bifurcation. The branches of periodic orbits are replaced by several explicitly computed limit cycles.

The double-gyre circulation: A different rung of the hierarchy

Another “intermediate” model of the double-gyre circulation: slightly different physics, higher resolution – down to 10 km in the horizontal and more layers in the vertical, much larger domain, …

Bo Qiu, U. of Hawaii, pers. commun., 1997



Quasi - geostrophic model

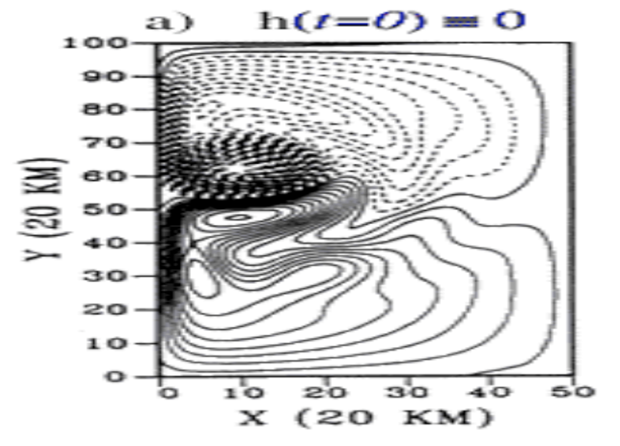
2.5-layer model

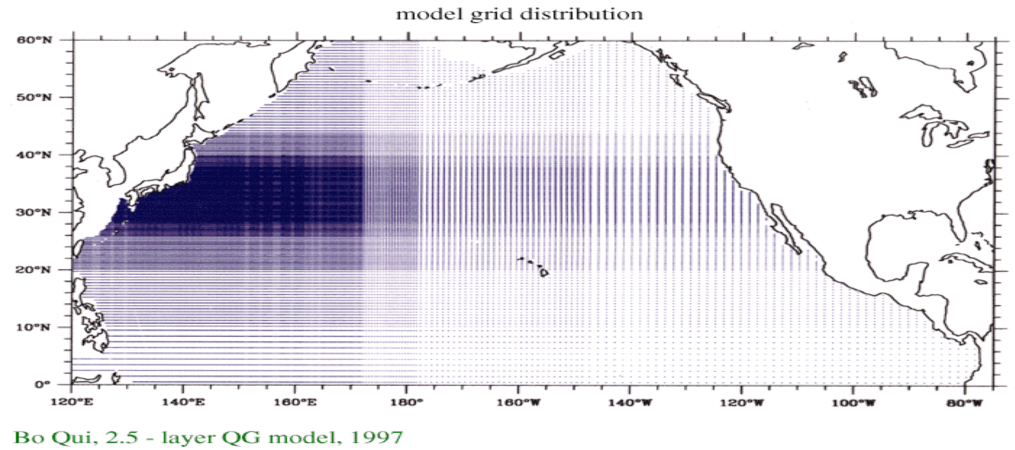
$$
\begin{aligned}
\frac{\partial}{\partial t}(\nabla^2 h_1 &- \lambda_1^2(h_1 - h_2)) + \beta\frac{\partial h_1}{\partial x} = -\frac{g'}{f_0}J[h_1, \nabla^2 h_1 - F_1(h_1 - h_2)] \\
&+ A_h\nabla^4 h_1 - C\nabla^2(h_1 - h_2) + \frac{f_0}{\rho_0 g' H_1} curl\vec{\tau} \\
\frac{\partial}{\partial t}(\nabla^2 h_2 &- \lambda_2^2(h_2 - h_1)) + \beta\frac{\partial h_2}{\partial x} = -\frac{g'}{f_0}J[h_2, \nabla^2 h_2 - F_2(h_2 - h_1)] \\
&+ A_h\nabla^4 h_2 - C\nabla^2(h_2 - h_1) - R\nabla^2 h_2
\end{aligned}
$$

where

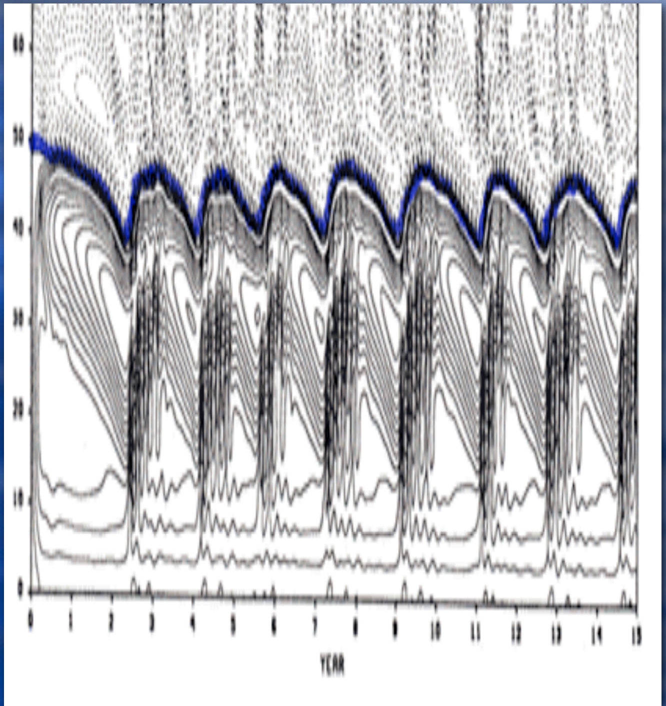
h_1, h_2:	height anomaly for upper and lower layer	**(stream functions)**
H_1, H_2:	mean height for upper and lower layer	
λ_1, λ_2:	Rossby radius of deformation	$\equiv \sqrt{h' H_1 / f_0^2}, \sqrt{h' H_2 / f_0^2}$
$\vec{\tau}$:	wind stress	
A_h:	viscosity coefficient	
C, R:	Rayleigh coefficient for interface and lower layer	
f_0, β:	Coriolis and beta parameters	
ρ_0, g':	mean density and reduced gravity	

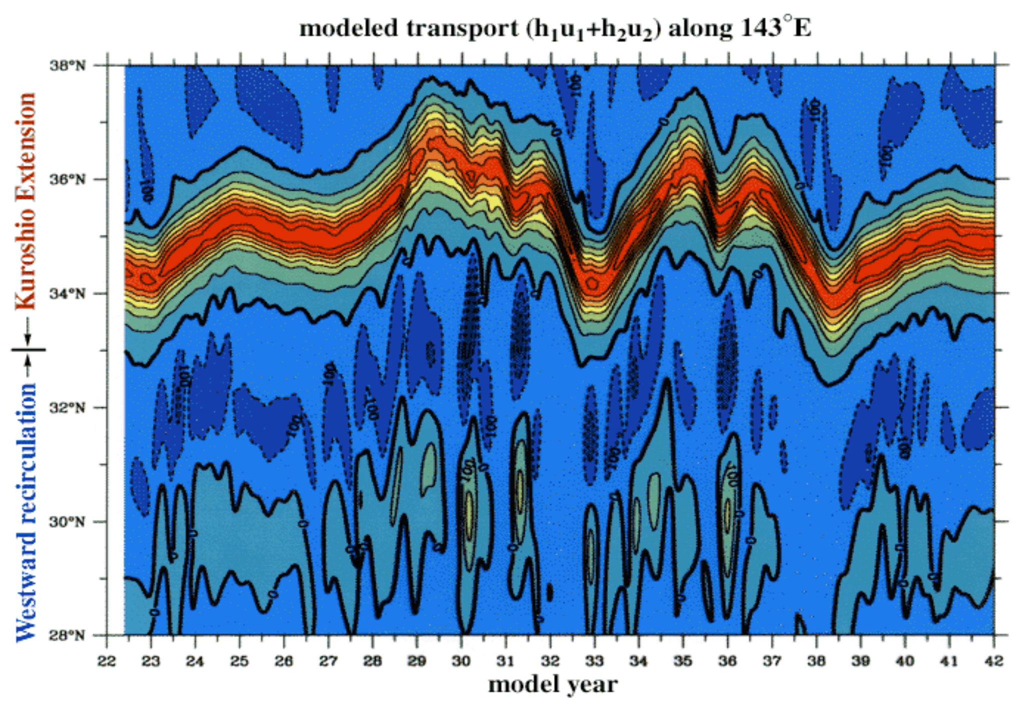
Model-to-model, qualitative comparison





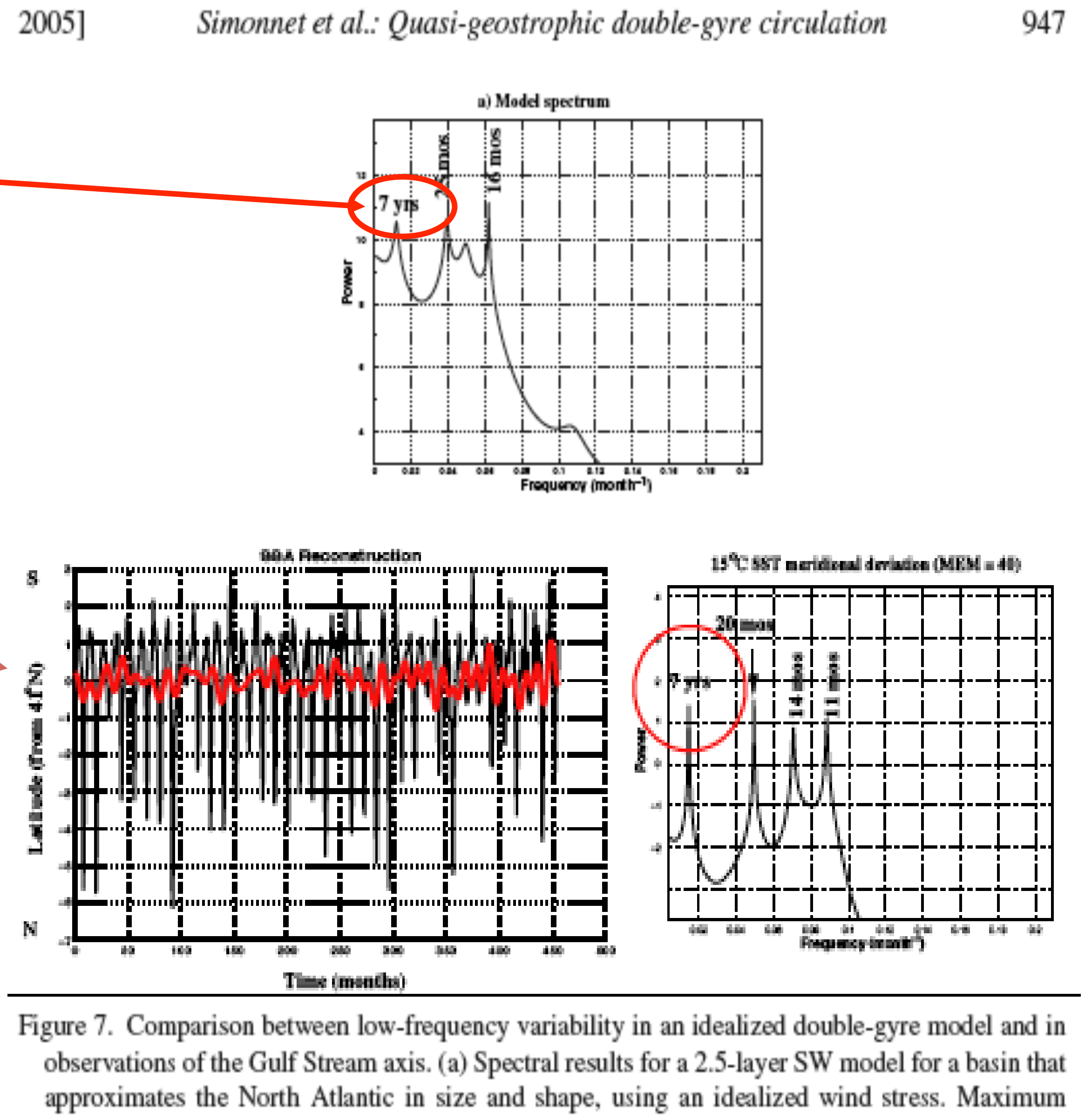
Bo Qui, 2.5 - layer QG model, 1997





Model vs. observations, a quantitative comparison

Spectra of
(a) kinetic energy of 2.5-layer shallow-water model in North-Atlantic–shaped basin; and
(b) Cooperative Ocean-Atmosphere Data Set (COADS) Gulf-Stream axis data



2005] *Simonnet et al.: Quasi-geostrophic double-gyre circulation* 947

Figure 7. Comparison between low-frequency variability in an idealized double-gyre model and in observations of the Gulf Stream axis. (a) Spectral results for a 2.5-layer SW model for a basin that approximates the North Atlantic in size and shape, using an idealized wind stress. Maximum

Simonnet, Ghil & Dijkstra
(*J. Mar. Res.*, 2005)

More spatio-temporal data

Multi-channel SSA analysis of the UK Met Office monthly mean SSTs for the century-long 1895–1994 interval

Marked similarity with the 7–8-year “gyre mode” of a full hierarchy of ocean models, on the one hand, and with the North Atlantic Oscillation (NAO), on the other: explanation?

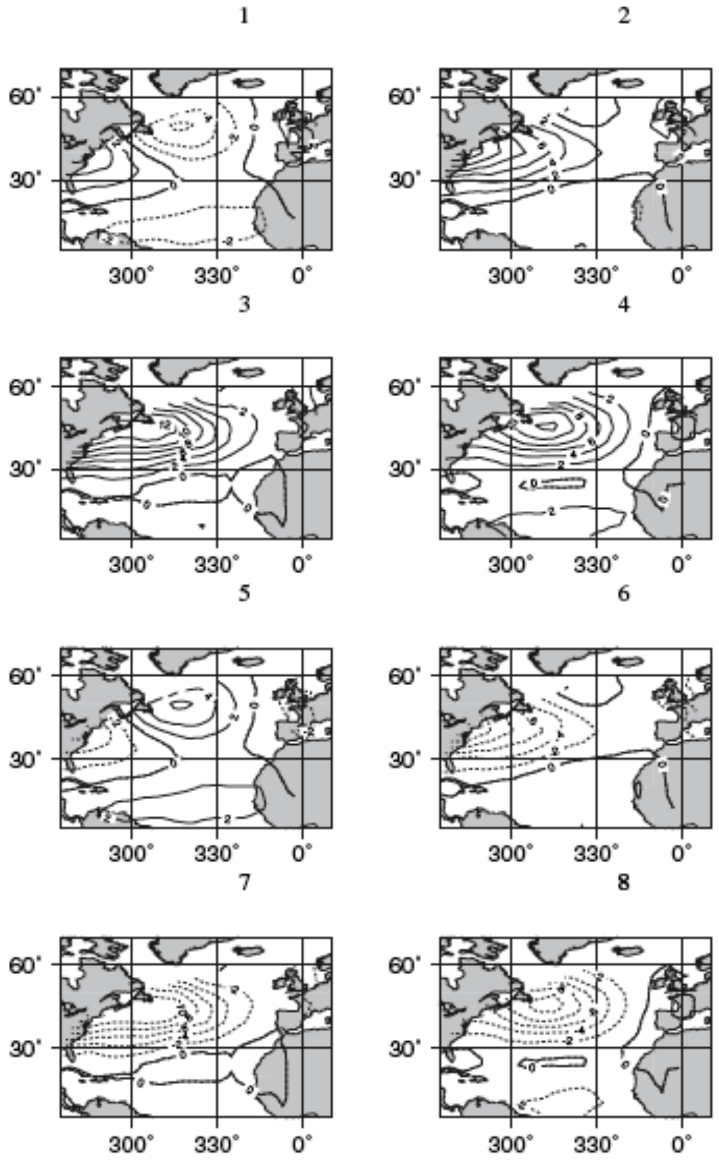


Figure 8. Phase composites of the reconstructed 7–8-year SST oscillation. The MSSA window length is 40 year and the contour interval is 0.02°C.

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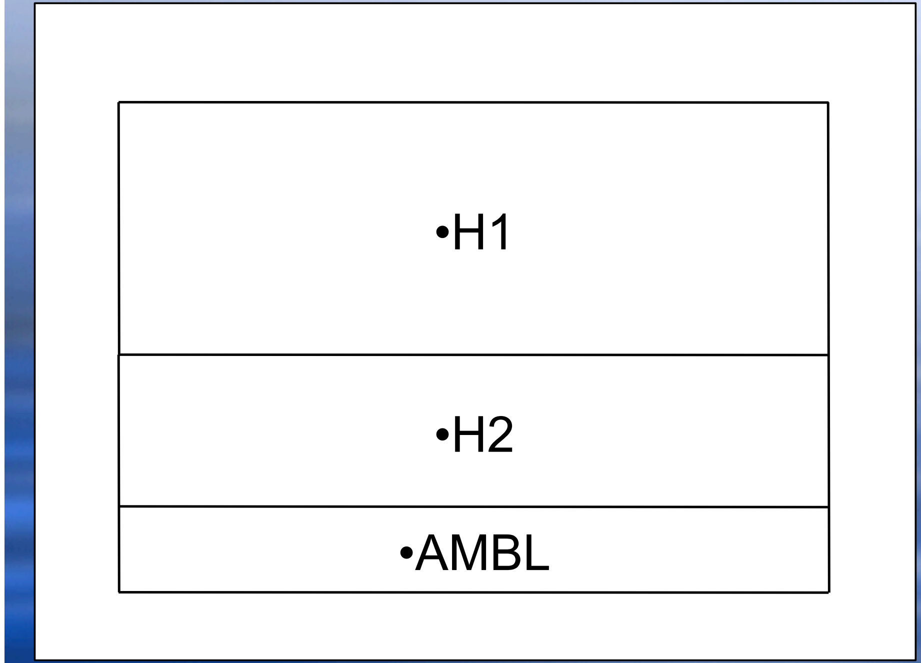
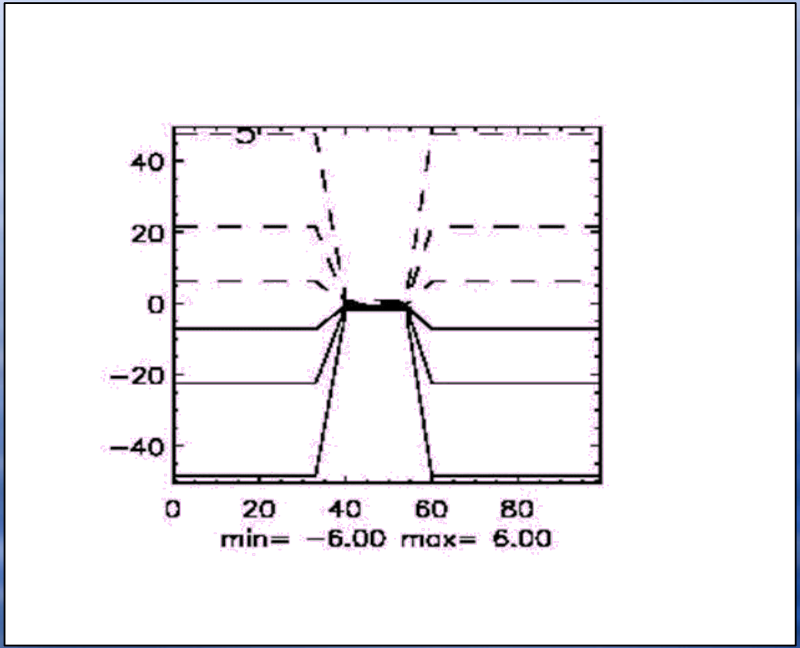
Atmospheric impact of mid-latitude SST anomalies: A highly contentious issue



•H1

•H2

•AMBL



- A quasi-geostrophic (QG) atmospheric model in a periodic β-channel, first barotropic (Feliks *et al.*, *JAS*, 2004; FGS'04), then baroclinic (FGS'07).
- Marine atmospheric boundary layer (ABL), analytical solution.
- Forcing by idealized oceanic SST front.

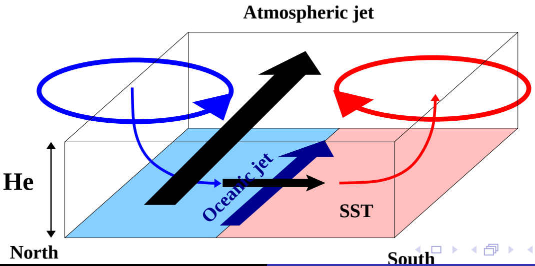
Ocean-atmosphere coupling mechanism (II)

Vertical velocity at the top of the marine ABL

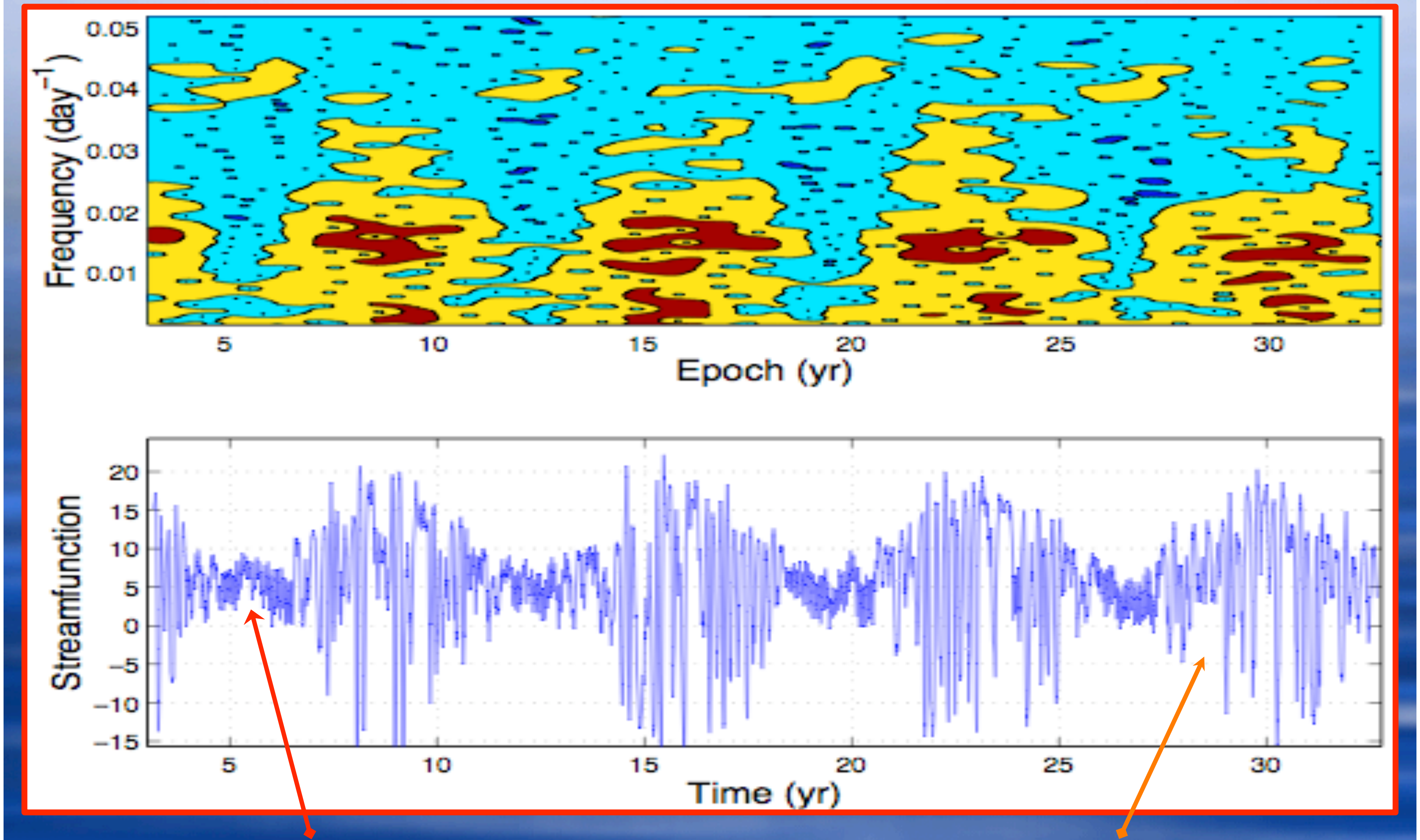
- The nondimensional $w(H_e)$ is given by

  $$w(H_e) = \left[\gamma\zeta_g - \alpha\nabla^2 T\right],$$

  with $\gamma = c_1(f_0L/U)(H_e/H_a)$ and $\alpha = c_2(g/T_0U^2)(H_e^2/H_a)$, where H_a is the layer depth of the free atmosphere ($\sim$ 10 km), and ζ_g the atmospheric geostrophic vorticity.
- Two components: one **mechanical**, due to the geostrophic flow ζ_g above the marine ABL and one **thermal**, induced by the SST front.



Evolutive spectral analysis



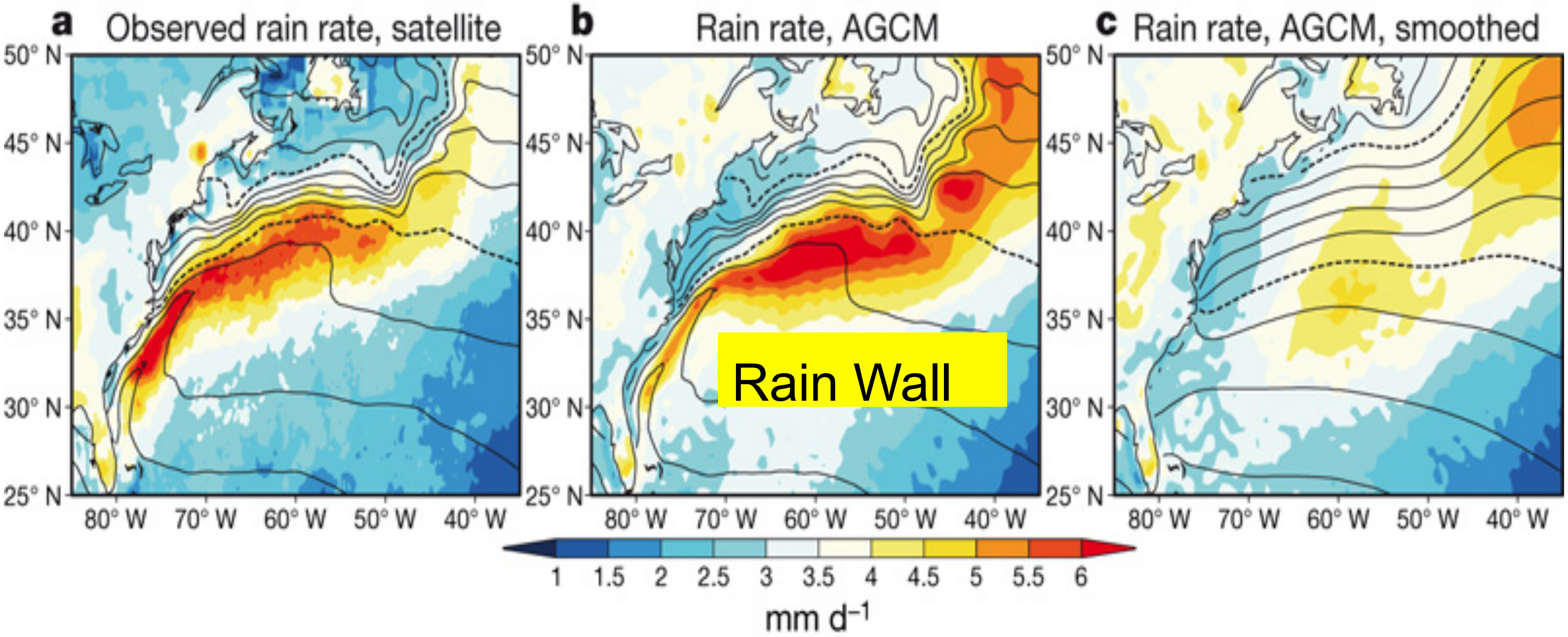
30-day oscillation

70-day oscillation

Precipitation effects:

sat. obs. (TRMM 3B43), ECMWF reanalyses,

and AFES (AGCM for the Earth Simulator, T239, 48 levels)



Minobe *et al*. (*Nature*, 2008):

smoothing the SST field suppresses the rain wall

IPCC-class GCM:
LMD-Z has zooming capability

Model set-up

– **19 levels, 3°x3°** outside the zoomed area and **0.5°x0.5°** inside it;

– **zoomed area** of (20° lat. x 40° long.), centered at (65°W,40°N);

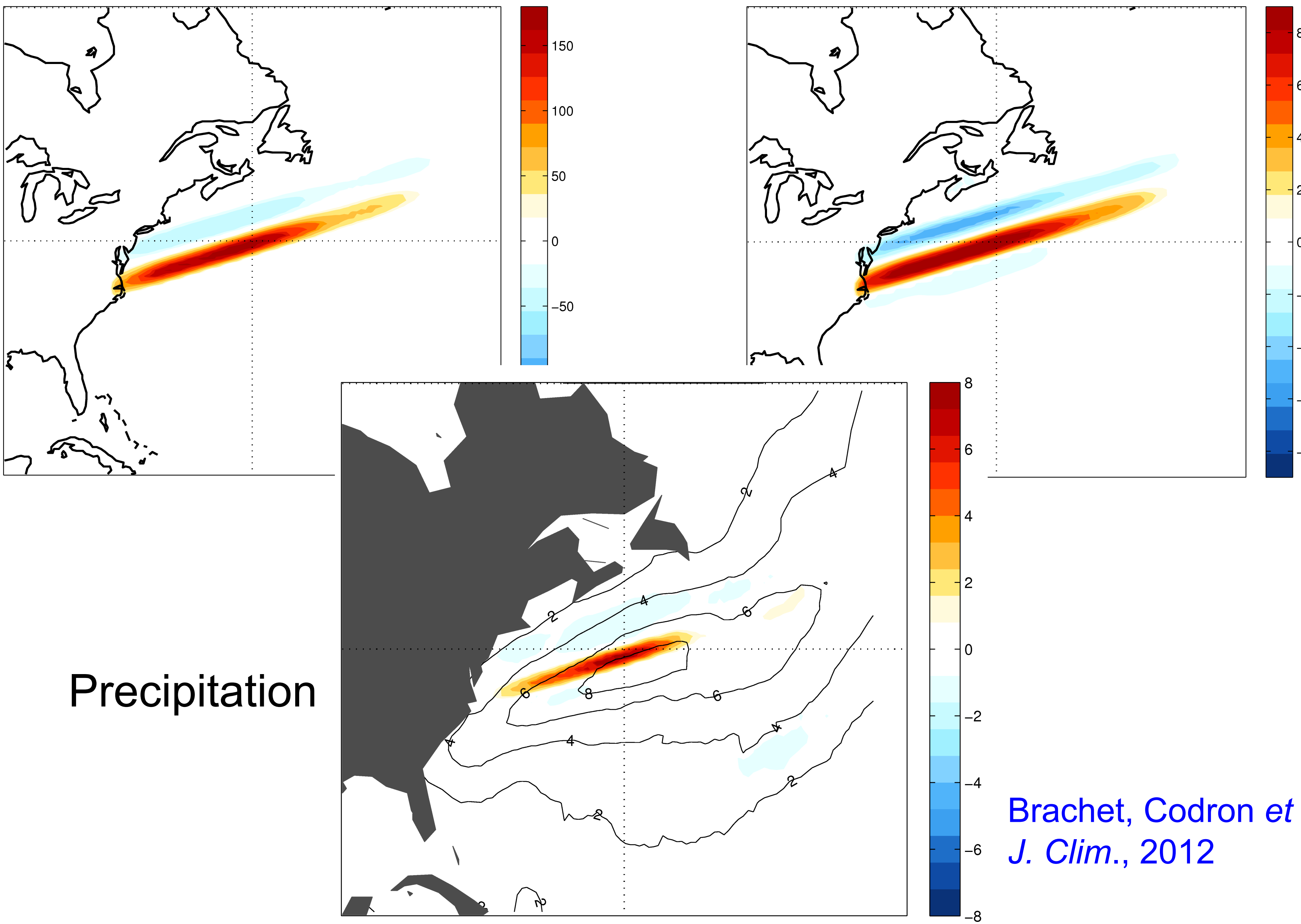
– **perpetual forcing**, corresponding to February 15.

3 simulations, 800-day long:

– a control simulation with the climatological SST field and no zoom;

– one with zoom and the climatological SST field still; and

– and one with zoom and a sharper SST front.


Evaporation

Sensible Heat Flux

Precipitation

Brachet, Codron *et al.*, *J. Clim.*, 2012

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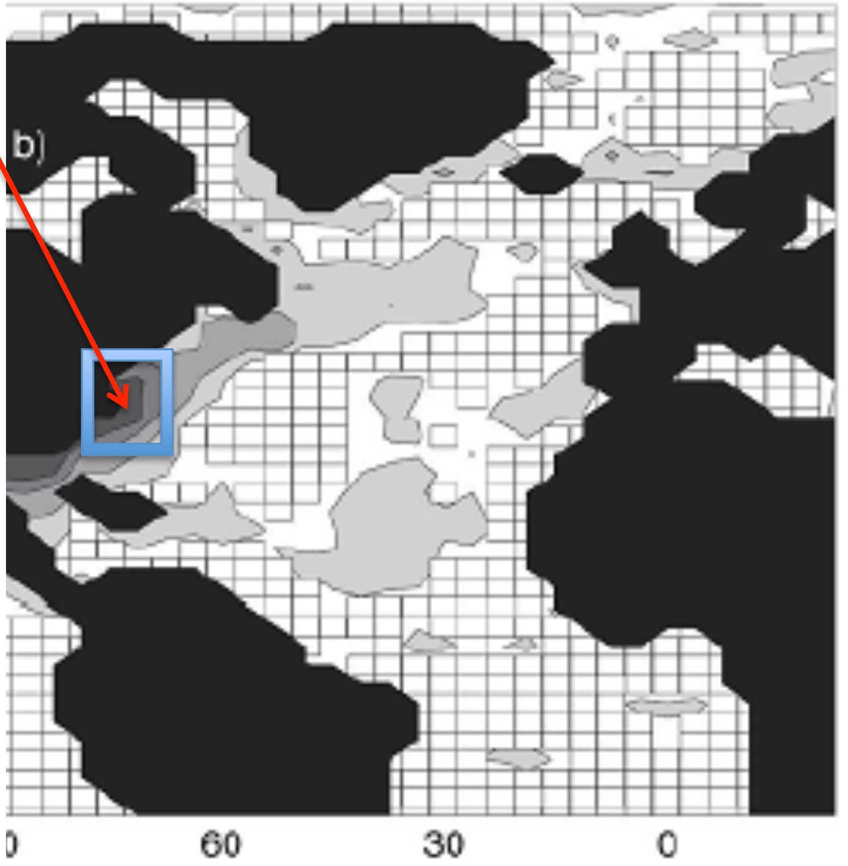
SST effects on NAO, via Granger causality

Q: Where does SST add information to knowledge of the NAO?

A: Just where you'd expect it!

Daily data from
50-yr simulation of
IPCC-class coupled GCM,
HadCM3

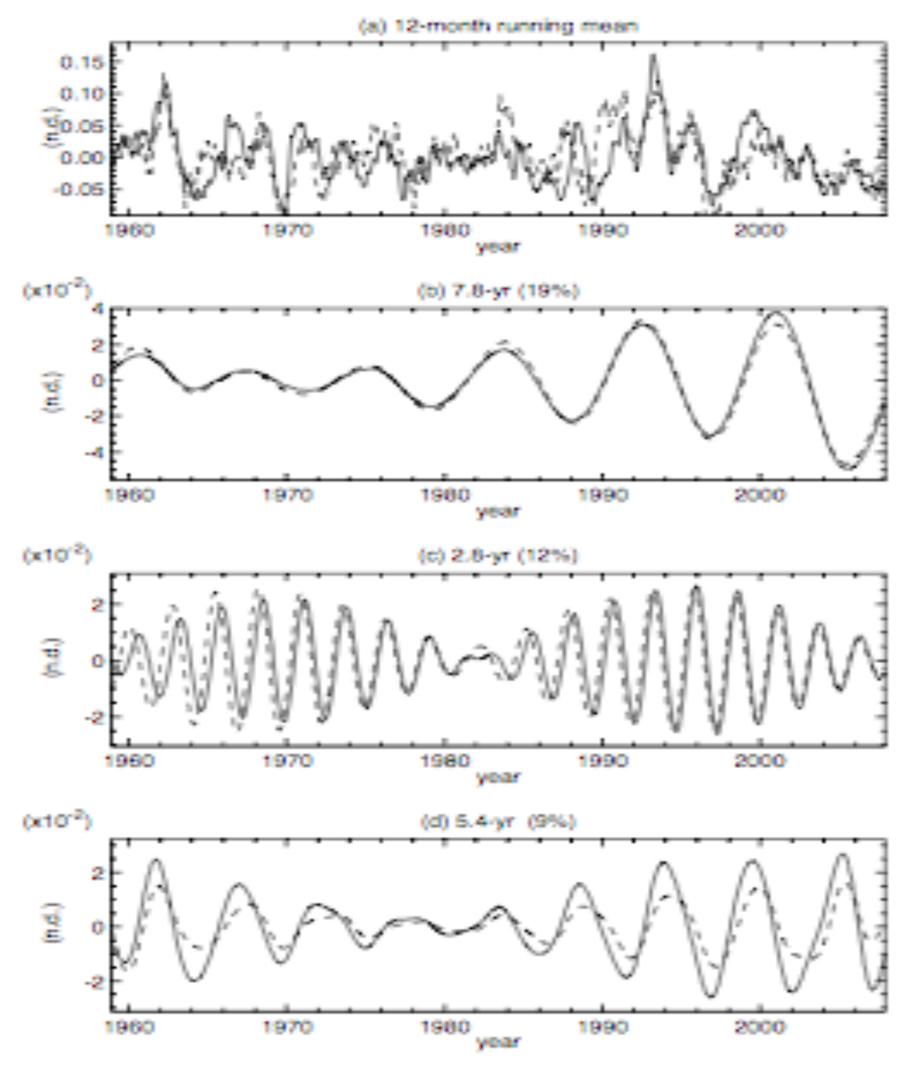


Mosedale, Stephenson, Collins & Mills
(*J. Climate*, 2006)

The 7–8-yr mode in atmospheric data
Likewise a contentious issue

Simulate atmospheric response to SODA data over the Gulf Stream region

- Use SST (–5 m) data from the SODA reanalysis (50 years)
- Use the FGS'07 QG model in periodic *β*-channel
  – baroclinic + marine ABL
- Figure shows NAO index:
  – simulated (solid)
  – observed (dashed)



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- The low-frequency variability of the double-gyre circulation
  - bifurcations in a toy model

    => multiple equilibria, periodic and chaotic solutions
  - some intermediate model results
- Atmospheric impacts
  - simple and intermediate models + GCMs
- Some data analysis – atmospheric and oceanic
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Time-dependent forcing, I

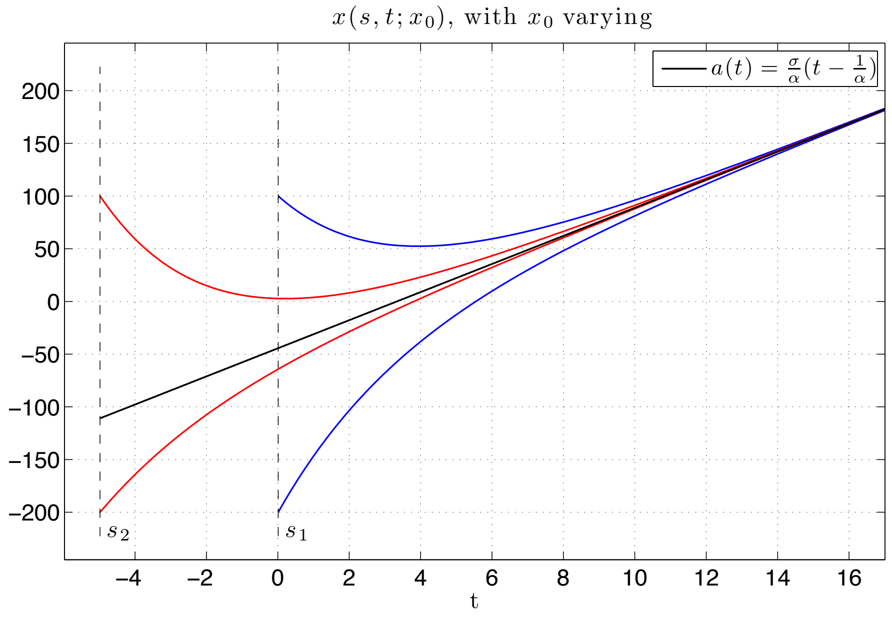
- Much of the theoretical work on the intrinsic variability of the wind-driven ocean circulation has been done with time-independent wind stress.
- To address truly coupled ocean–atmosphere behavior and climate change an important step is to examine time-dependent wind stress.
- The proper framework for doing so is the theory of non-autonomous and random dynamical systems (NDS and RDS).
- We do so here with a “toy” model given by the low-order truncation of the QG, equivalent-barotropic potential vorticity equation (PVE).
- The forcing is deterministic, aperiodic, and dominated by multi-decadal variability.

The pullback attractor of a linear, scalar ODE,

$$\dot{x} = -\alpha x + \sigma t, \quad \alpha > 0, \; \sigma > 0\,,$$

is given by

$$a(t) = \frac{\sigma}{\alpha}\left(t - \frac{1}{\alpha}\right).$$



Time-dependent forcing, II

The highly idealized, toy model of the QG, equivalent-barotropic PVE is given by the following system of four quadratically nonlinear ODEs:

$$\dot{\psi}_1 + L_{11}\psi_1 + L_{13}\psi_3 + B_1(\Psi,\Psi) = W_1(t),$$
$$\dot{\psi}_2 + L_{22}\psi_2 + L_{24}\psi_4 + B_2(\Psi,\Psi) = W_2(t),$$
$$\dot{\psi}_3 + L_{33}\psi_3 + L_{31}\psi_1 + B_3(\Psi,\Psi) = W_3(t),$$
$$\dot{\psi}_4 + L_{44}\psi_4 + L_{42}\psi_2 + B_4(\Psi,\Psi) = W_4(t);$$

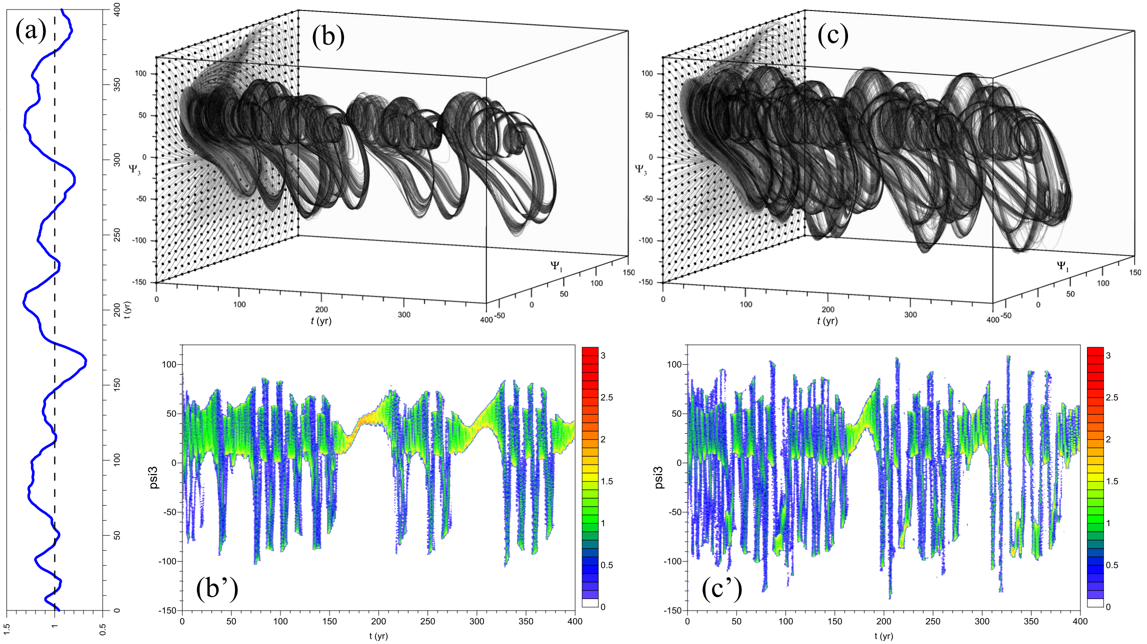
where Ψ denotes the vector $(\psi_1, \psi_2, \psi_3, \psi_4)$ and the bilinear terms B_i are given by

$$\begin{aligned}
B_1(\Psi,\Psi) &= 2J_{112}\psi_1\psi_2 + 2J_{114}\psi_1\psi_4 + 2J_{134}\psi_3\psi_4,\\
B_2(\Psi,\Psi) &= J_{211}\psi_1^2 + J_{222}\psi_2^2 + J_{233}\psi_3^2\\
&\quad + 2J_{213}\psi_1\psi_3 + 2J_{224}\psi_2\psi_4,\\
B_3(\Psi,\Psi) &= 2J_{314}\psi_1\psi_4 + 2J_{323}\psi_2\psi_3 + 2J_{334}\psi_3\psi_4,\\
B_4(\Psi,\Psi) &= J_{411}\psi_1^2 + J_{422}\psi_2^2 + J_{433}\psi_3^2 + J_{444}\psi_4^2\\
&\quad + 2J_{413}\psi_1\psi_3 + 2J_{424}\psi_2\psi_4.
\end{aligned}$$

Time-dependent forcing, III

- The quadratic terms are conservative and the linear terms are weakly dissipative, while the system is unstable for reasonable parameter values.
- For autonomous systems, we know that these properties can lead to chaotic solutions that live on a strange attractor.
- Here they lead to the existence of a pullback attractor (PBA).

Time-dependent forcing at far
left + ensemble of 644 orbits
starting from the same subset Γ but
with different parameter values:
left panels, close to periodic;
right panels, fully chaotic;
lower panels, time series of ψ_3.
Time interval is 400 years.

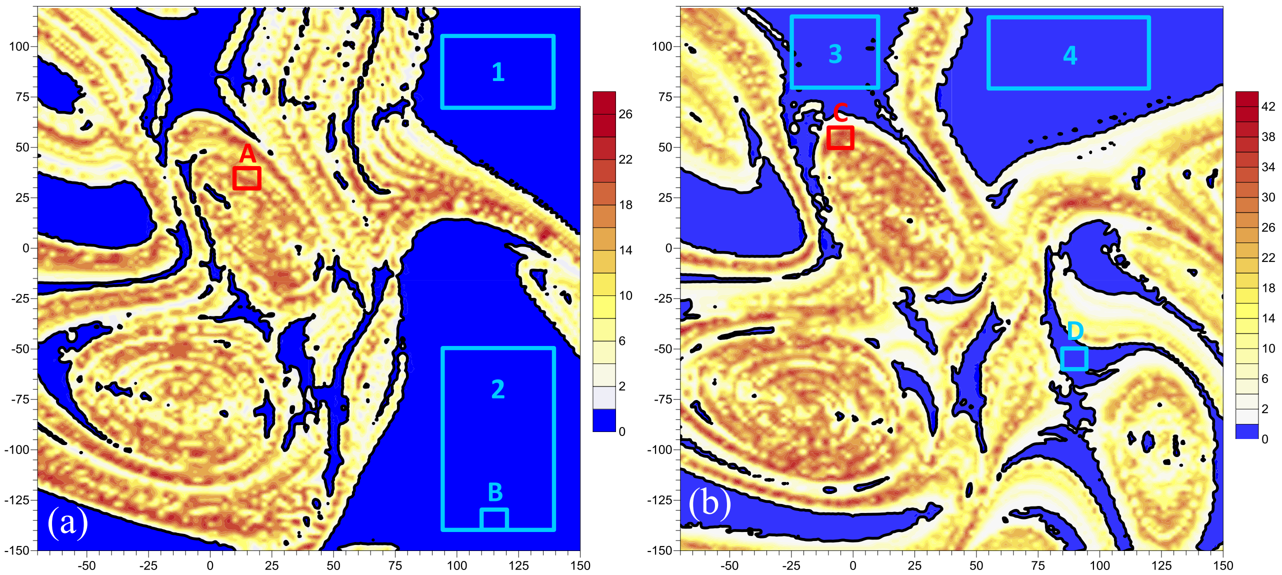


Pierini, Chekroun & Ghil
(*J. Climate*, 2016)

Time-dependent forcing, IV

- There are strong numerical indications, along with theoretical justifications, that multiple PBAs are present within a global attractor.
- Moreover, preliminary numerical results suggest that the basin boundaries between two attractors are fractal.

Measure of divergence of trajectories for each initial point in the (ψ_1, ψ_3)-plane in the remote past: blue indicates stability; parameter values (left) and (right) are the same as in the previous figure.



Pierini, Chekroun & Ghil
(*J. Climate*, 2016)

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Concluding remarks

What do we know?

- There's an NAO, & it's important.
- It has decadal variability (7–8 yr).
- An oscillatory mode, albeit weak, can help prediction.
- Time-dependent forcing helps understand the coupled system.

What do we know less well?

- How does the climate system really work?
- Is it the tail that wags the dog —
  i.e., weather noise that drives a passive ocean?
- Or does the dog bite its tail –
  i.e., coupled O–A modes of decadal variability?
- Or does the old dog ocean plain wag its tail, the atmosphere?

What to do?

- Work the model hierarchy, and the observations!
- Explore further non-autonomous and randomly driven models,
  on the way to fully coupled ones.

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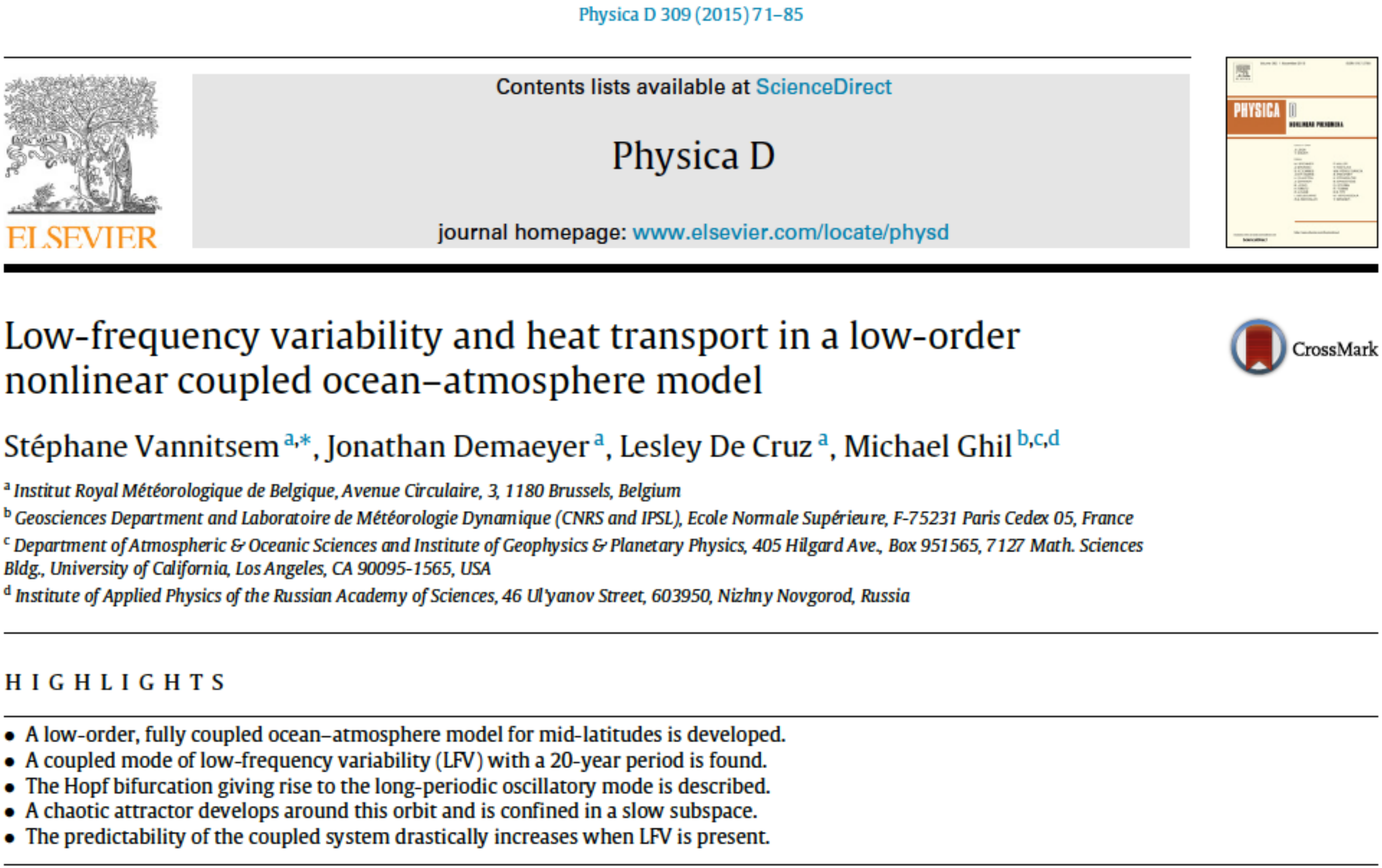
Coupled Ocean–Atmosphere Models, I



Physica D 309 (2015) 71–85

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journal homepage: www.elsevier.com/locate/physd

Low-frequency variability and heat transport in a low-order nonlinear coupled ocean–atmosphere model

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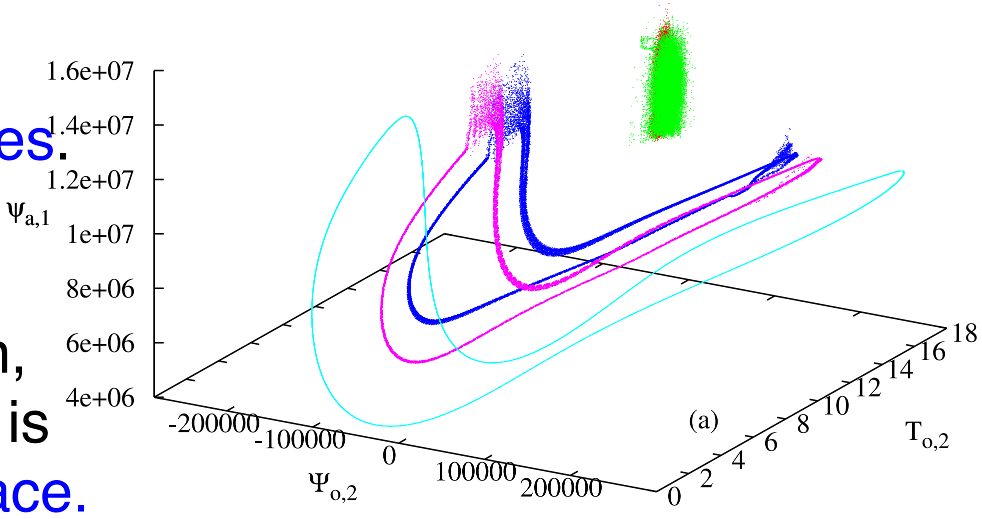
[d] *Institute of Applied Physics of the Russian Academy of Sciences, 46 Ul'yanov Street, 603950, Nizhny Novgorod, Russia*

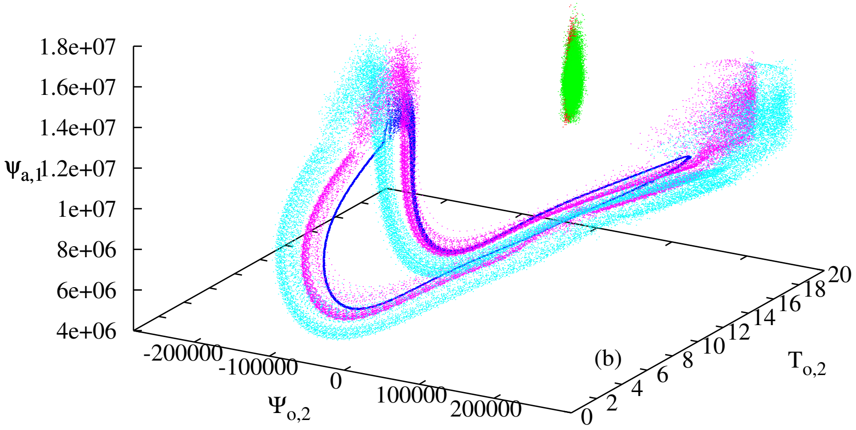
HIGHLIGHTS

- A low-order, fully coupled ocean–atmosphere model for mid-latitudes is developed.
- A coupled mode of low-frequency variability (LFV) with a 20-year period is found.
- The Hopf bifurcation giving rise to the long-periodic oscillatory mode is described.
- A chaotic attractor develops around this orbit and is confined in a slow subspace.
- The predictability of the coupled system drastically increases when LFV is present.

Coupled Ocean–Atmosphere Models, II

- Low-order model: 20 atmospheric ODEs + 16 oceanic ones = 36 modes.
- Hopf bifurcation leads to a bidecadal orbit.
- This orbit gives rise, in turn, to a strange attractor that is confined to a slow subspace.
- This subspace involves both atmospheric and oceanic modes.
- As forcing increases or damping decreases, the dimension of the attractor increases and it gets noisier.





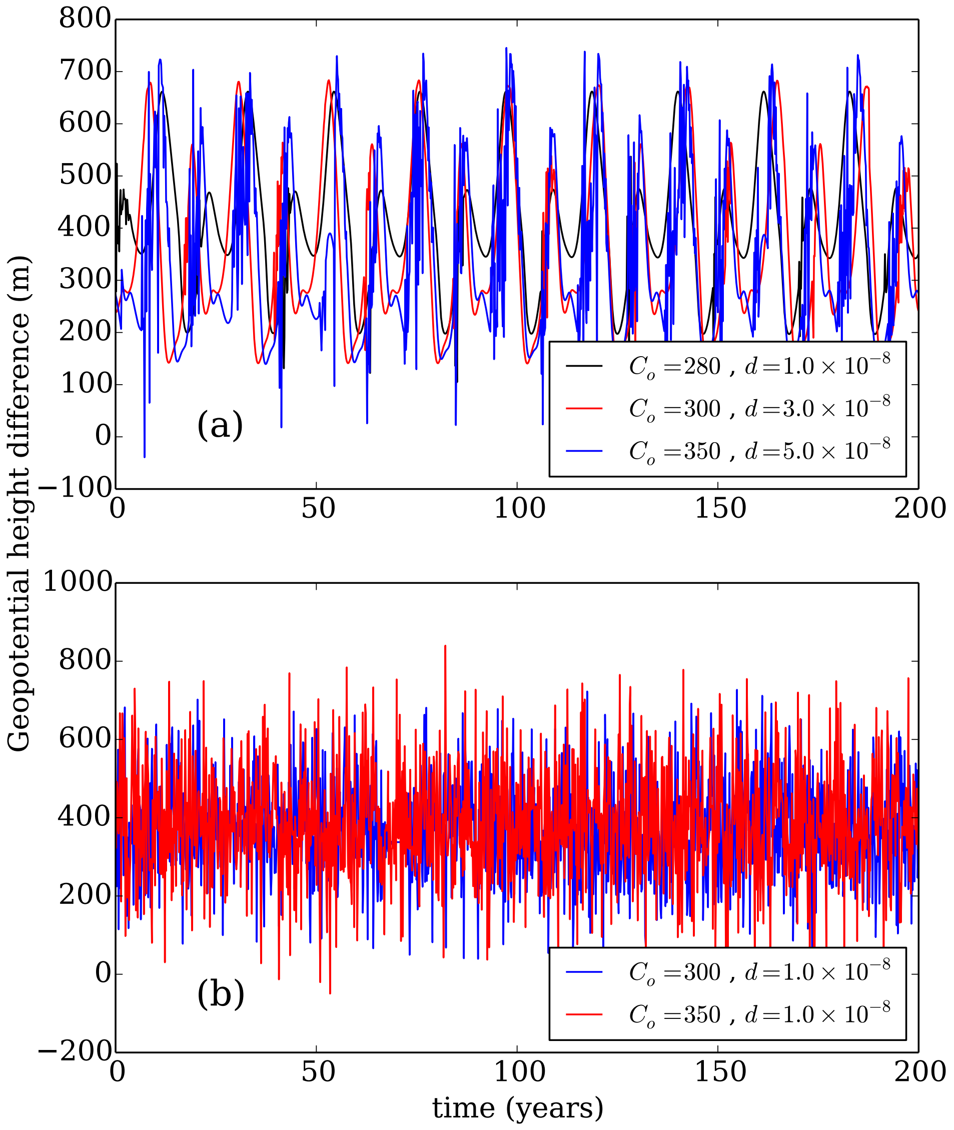
Coupled Ocean–Atmosphere Models, III

For certain parameter values (*), solutions are chaotic but smooth: low-frequency variability (LFV) dominates ➔ atmospheric predictability is high.

(*) i.e., low forcing or high damping

For other parameter values (**), fluctuations are rapid and large: solutions lie far from a slow attractor ➔ atmospheric predictability is low.

(**) i.e., high forcing or low damping



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Some references

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Reserve slides

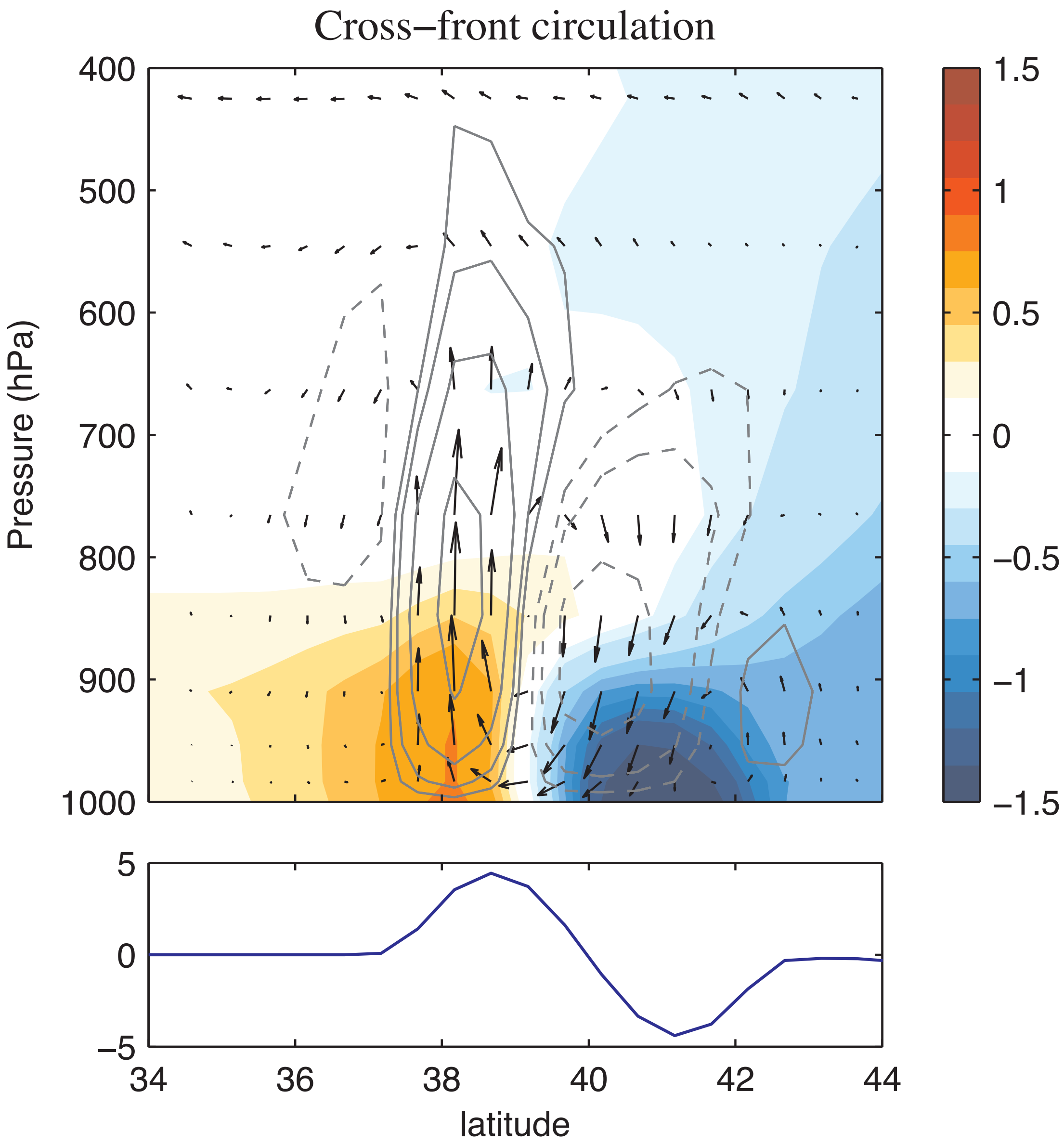
Cross-front circulation in the LMDz

Latitude–height cross section of the time-mean STR–CTL differences.

Temperature (color), vertical speed (contours), arrows scaled to match divergence.

The GCM simulations agree with the FGS theory.

An additional detail is that the ascent over the warm anomalies is deeper and more intense than the descent over the cold ones.



Brachet, Codron et al. (*J. Clim.*, 2012)

“Limited-contour” analysis for atmospheric low-frequency variability

10-day sequences of

subtropical jet paths:

blocked vs. zonal

flow regimes

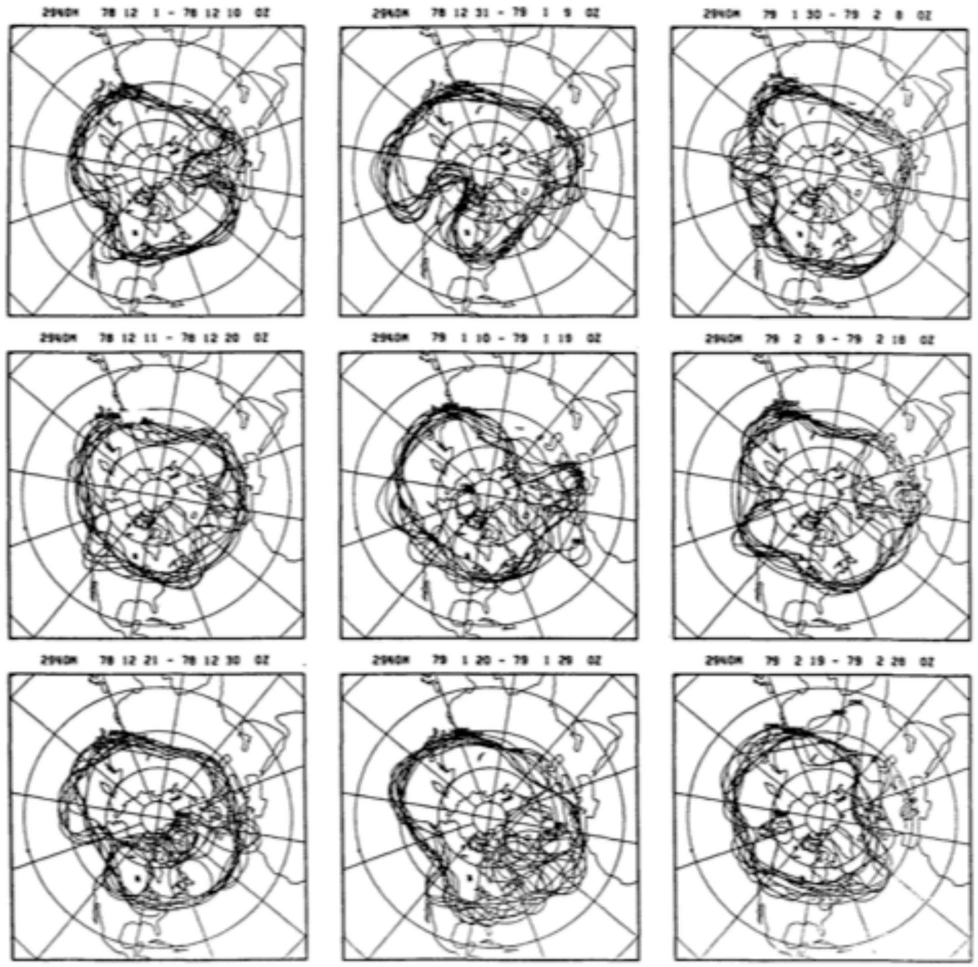


FIG. 1. Limited contour analysis of Northern Hemisphere (NH) flows. Daily contours of a prescribed height (2940 m in this case—roughly corresponding to the jet axis) are superimposed for successive 10-day intervals during NH winter 1978/79. Persistence is illustrated by some of the panels (see text for details).

Kimoto & Ghil, *JAS*, 1993a

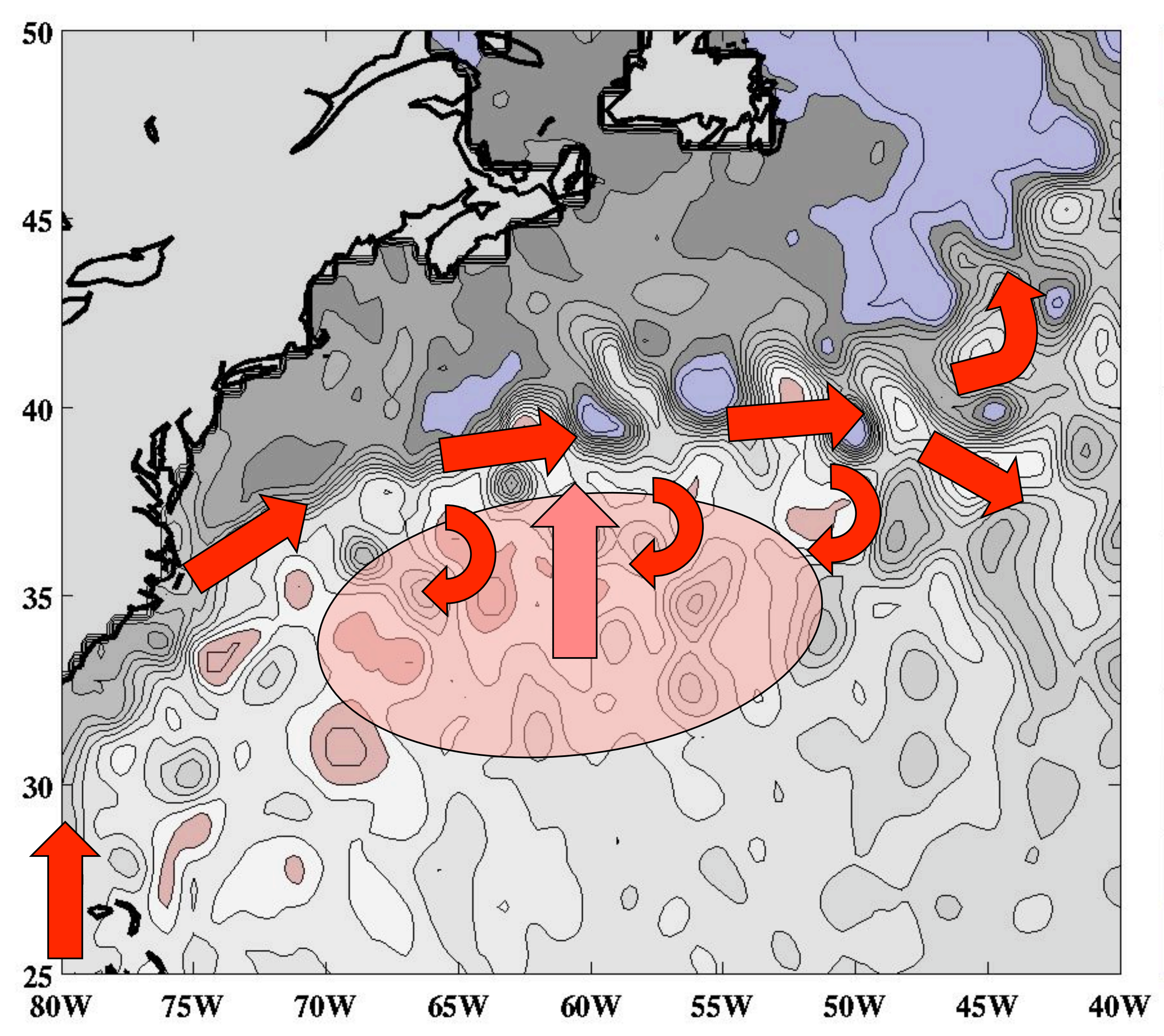
Western North Atlantic Circulation

Florida Current brings warm water north

Gulf Stream separates & recirculates

Recirculation creates heat reservoir

Heat fluxed to atmosphere

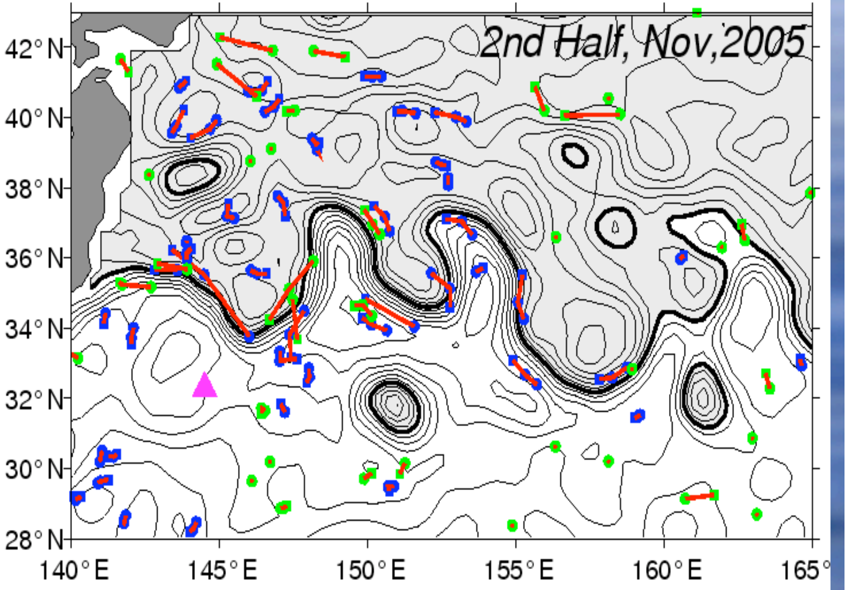


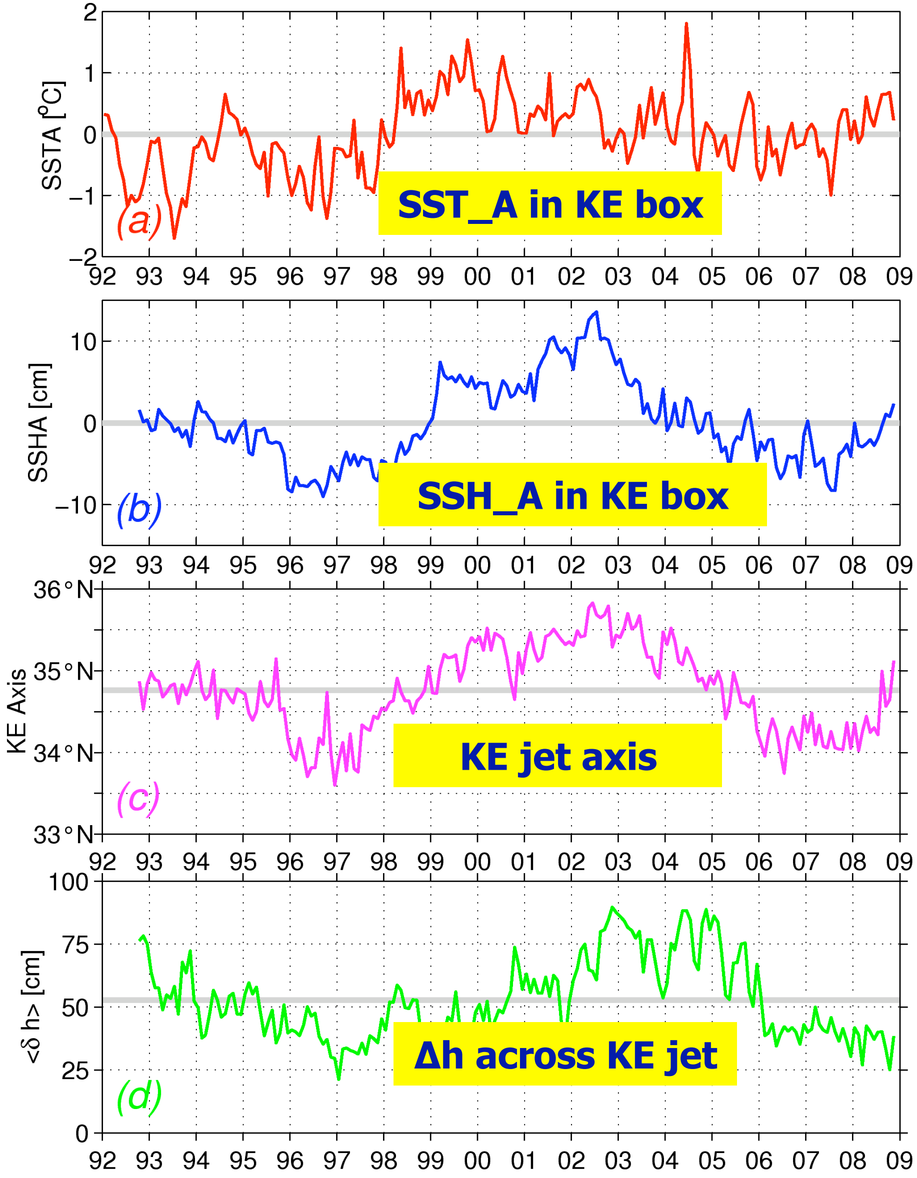
Kelly, Jan 2009

Applied Physics Laboratory
University of Washington

Kuroshio Extension (KE) box

SST anomalies are largely caused by strength changes of the KE jet





Courtesy of Bo Qiu, Jan. '09

Some references

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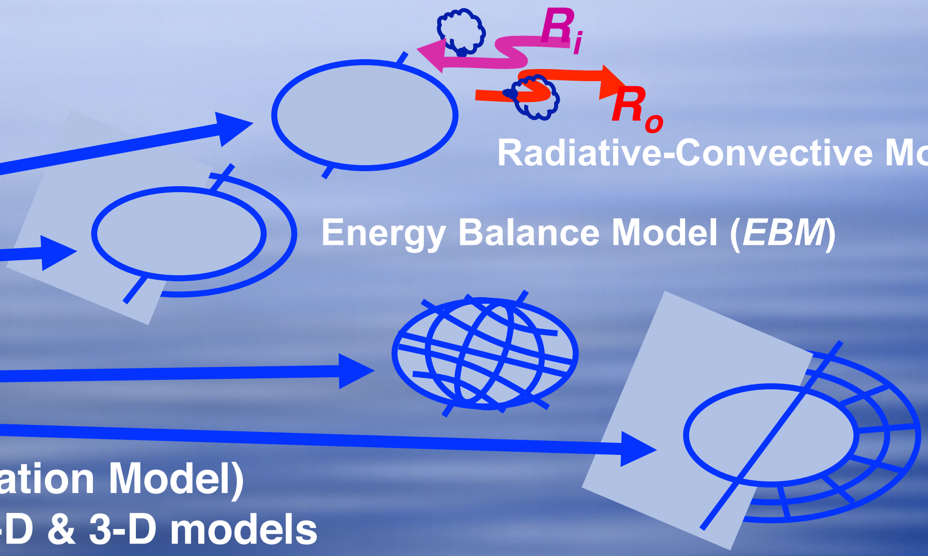
Climate models (atmospheric & coupled) : A classification

- *Temporal*
  - stationary, (quasi-)equilibrium
  - transient, climate variability
- *Space*
  - 0-D (dimension 0)
  - 1-D
    - vertical
    - latitudinal
  - 2-D
    - horizontal
    - meridional plane
  - 3-D, GCMs (General Circulation Model)
  - Simple and intermediate 2-D & 3-D models



Radiative-Convective Model(*RCM*)

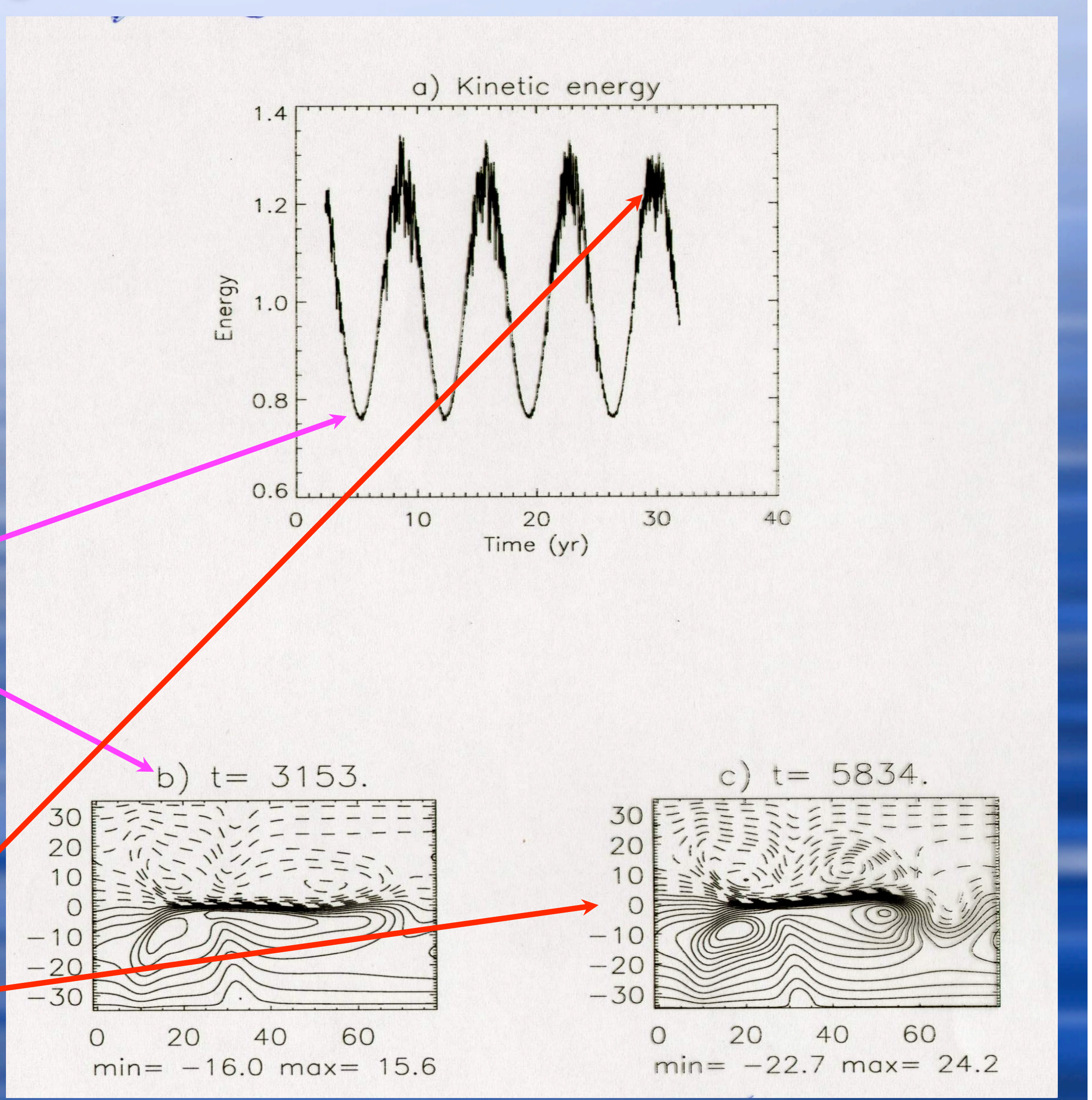
Energy Balance Model (*EBM*)

- *Coupling*
  - Partial
    - unidirectional
    - asynchronous, hybrid
  - Full

Hierarchy: from the simplest to the most elaborate,
iterative comparison with the observational data

Forced 7-year cycle in the FGS'04 model

Slow amplitude modulation of 1 ^{0}C in the SST front



Low-energy phase

High-energy phase

Spin-up of atmospheric jet

SST front:

L_{oc} = 600 km,

ΔT = 3.5 ^{0}C,

d = 50 km

Atmospheric jet

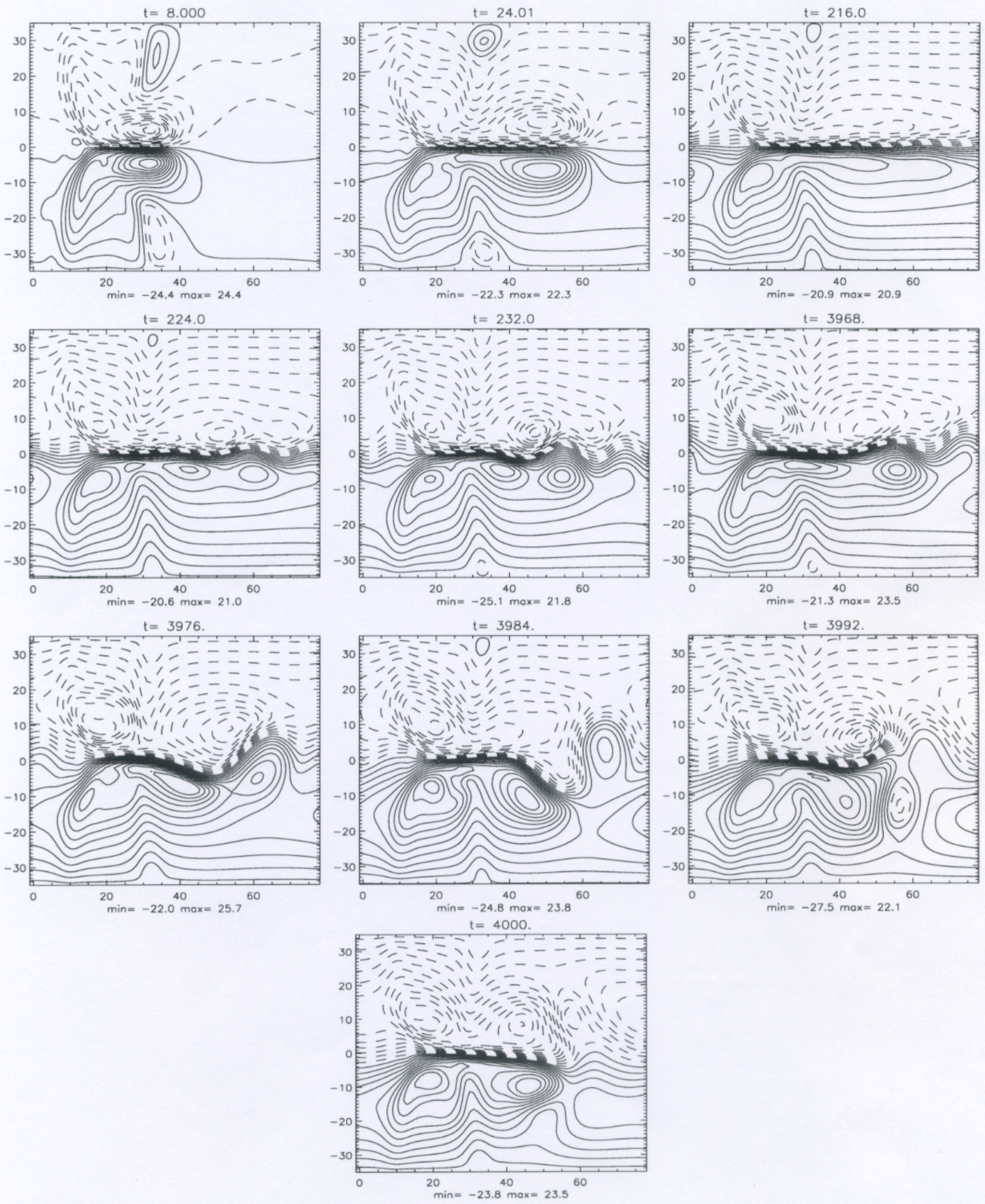
spins up from

L_a = 2000 km to

L_a = 4000 km, much

greater speed and

strong recirculation



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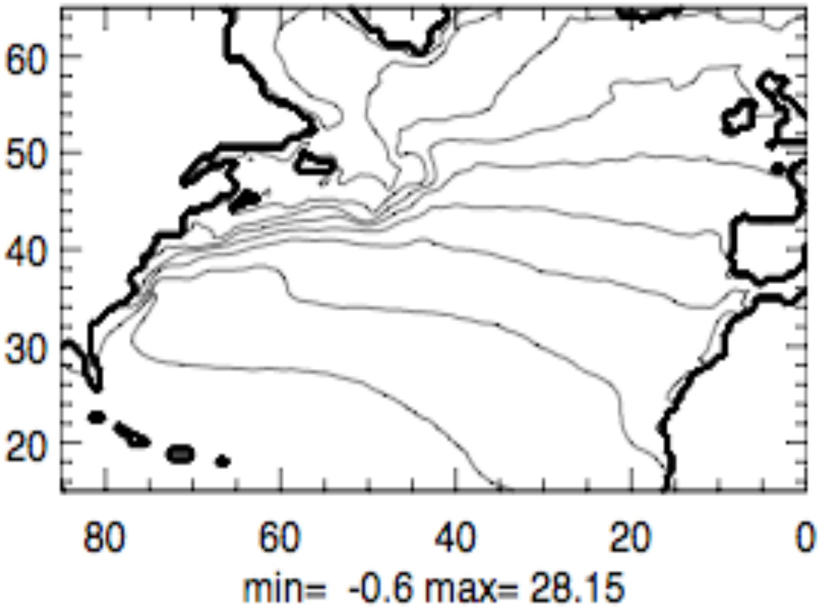
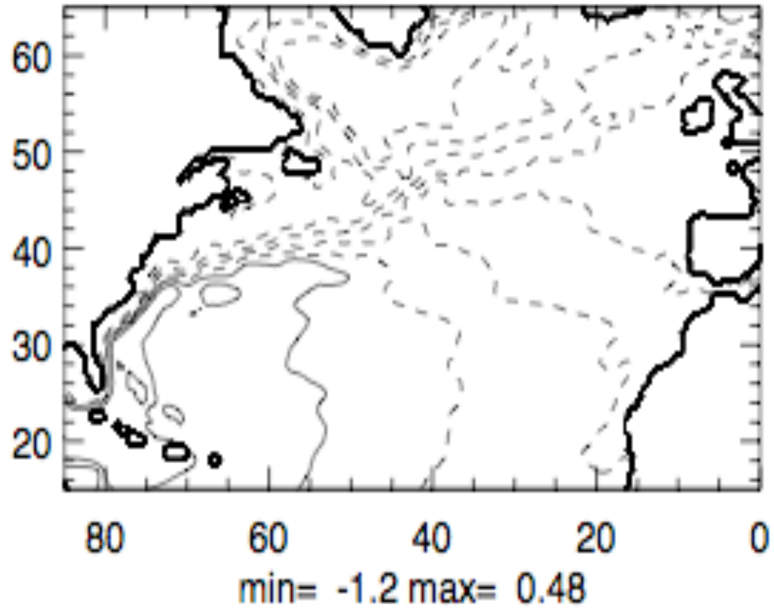
The 7–8-yr mode in oceanic data – I:
A still contentious issue

Mean SST field



Mean SSH field

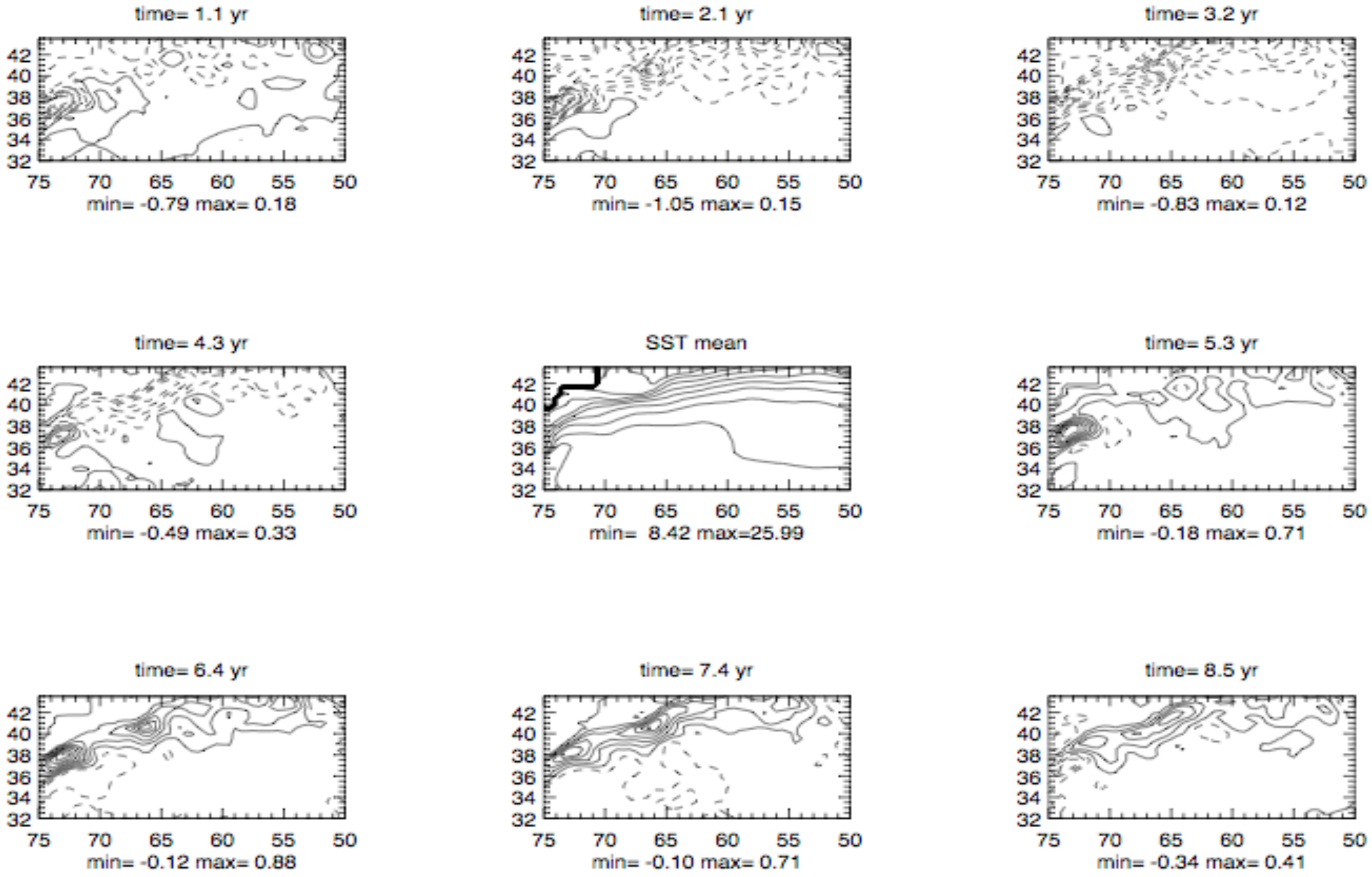


Simple Ocean Data Assimilation (SODA) reanalysis:

- Western North-Atlantic "rectangle" (28 N–42.5 N, 80 W-67.5 W);
- 50 years = Jan. 1958–Dec. 2007 (Carton and Giese, *MWR*, 2008).

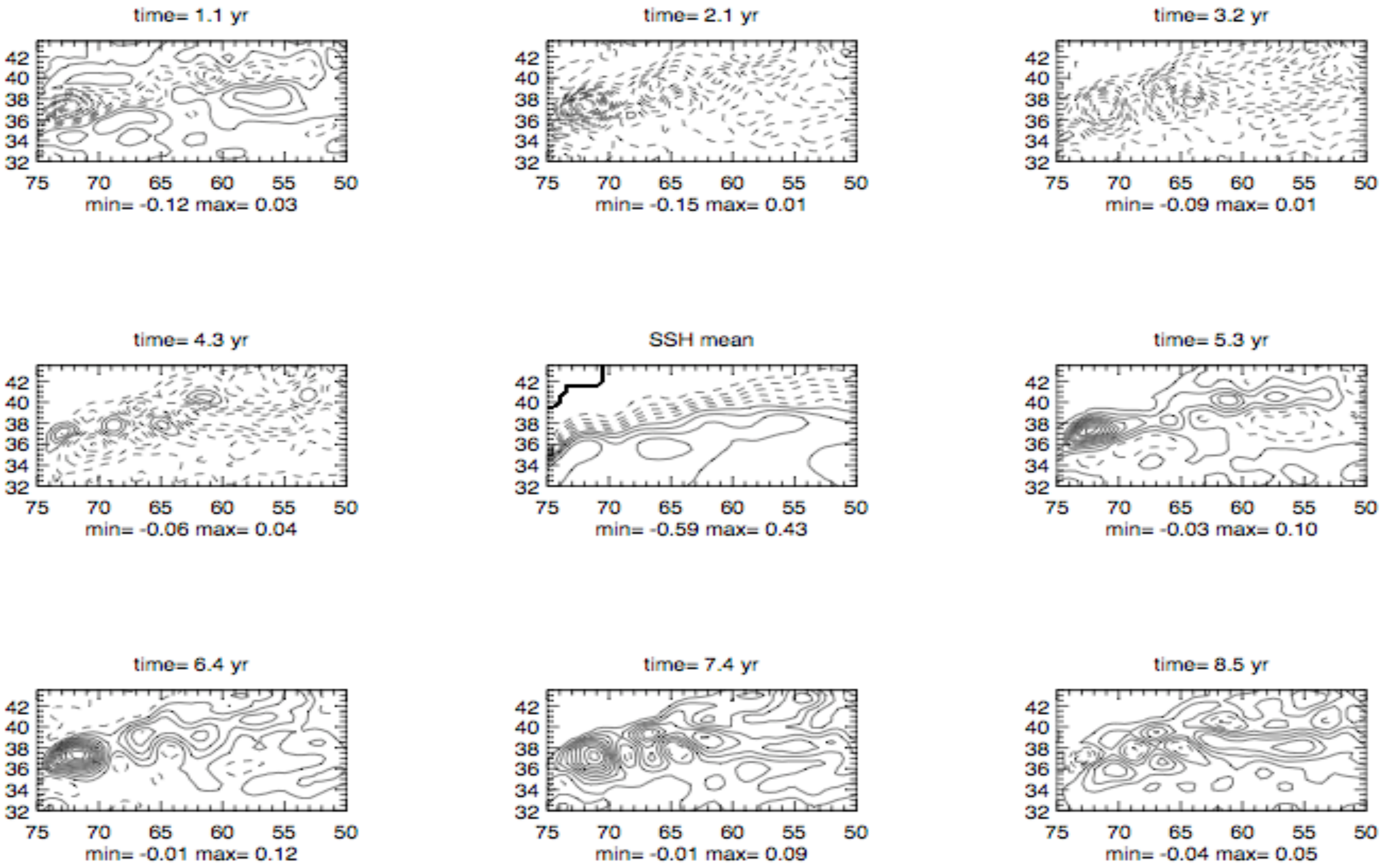
The 7–8-yr mode in oceanic data – II:

The SST field



The 7–8-yr mode in oceanic data – III:

The SSH field



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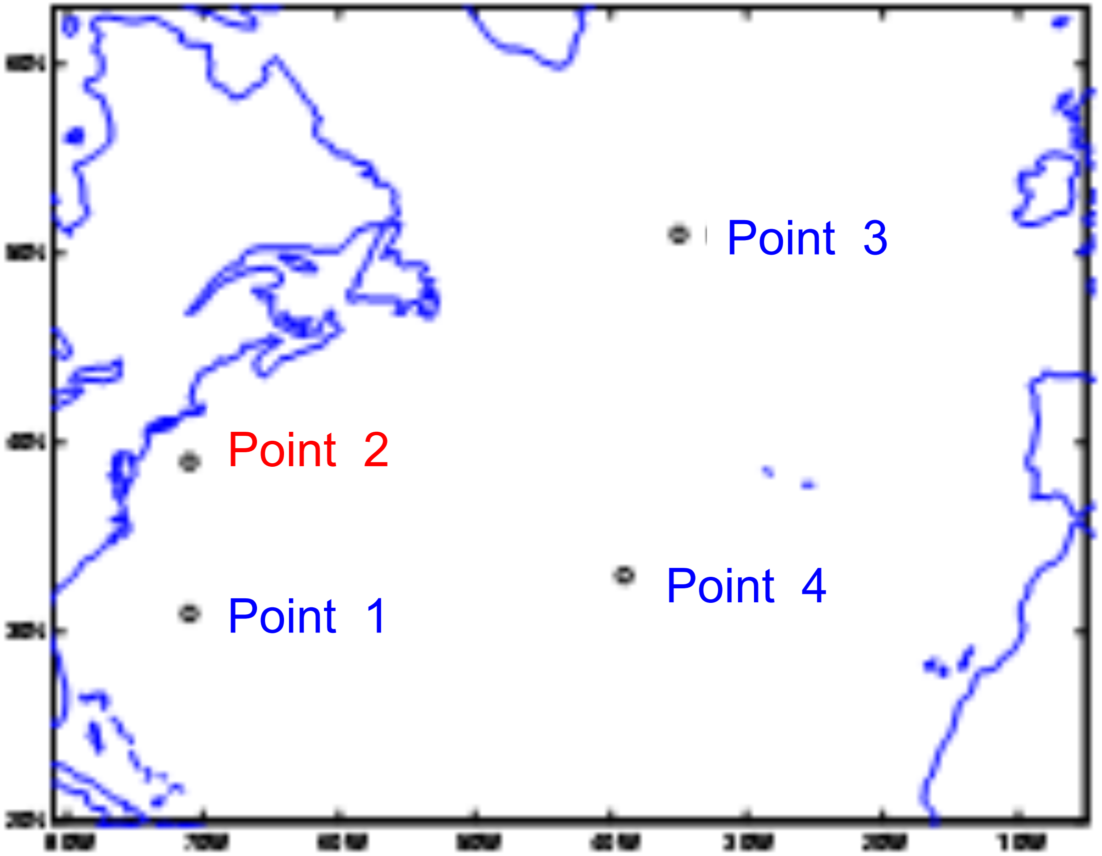
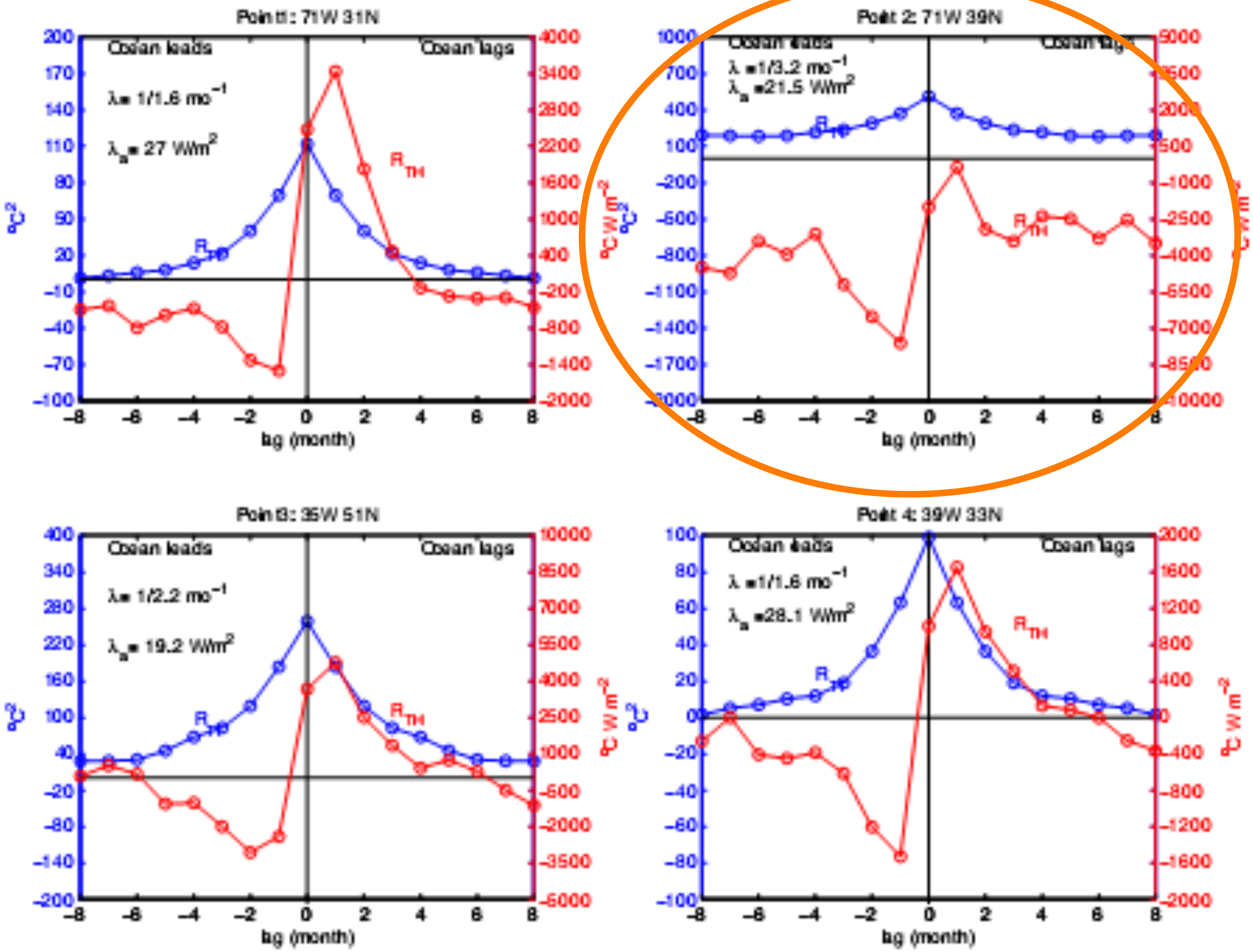
Study of SST and air-sea heat fluxes at 4 points



Courtesy of F. Codron

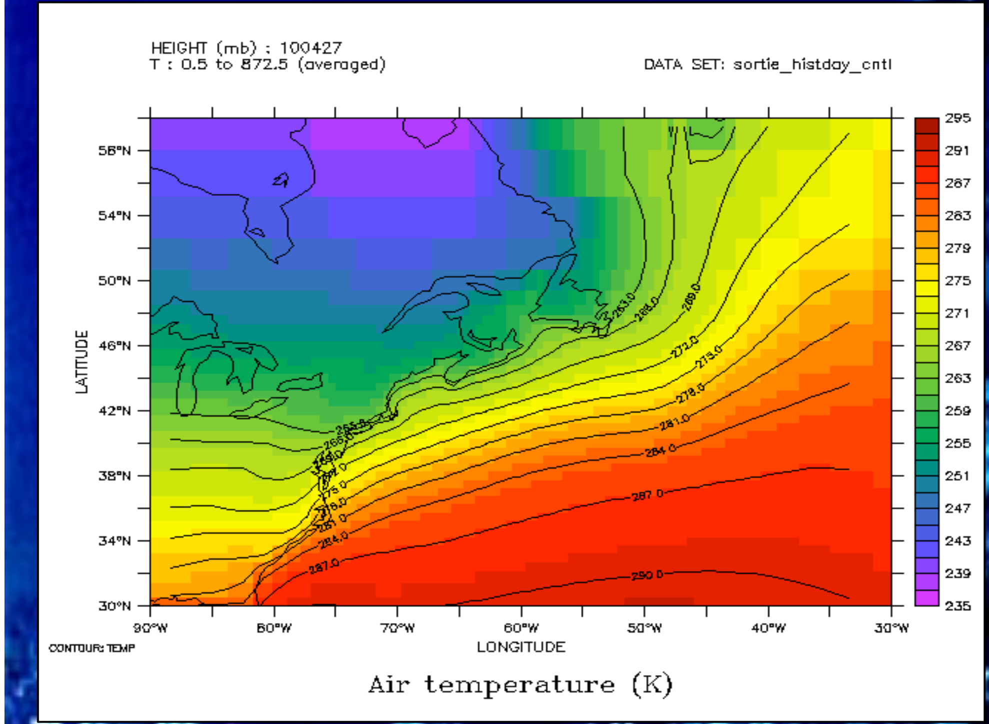
Lagged regressions: PC1 SST; PC1 SST *vs.* PC1 Air-Sea Flux

Exception for Point 2 (Gulf Stream Region)



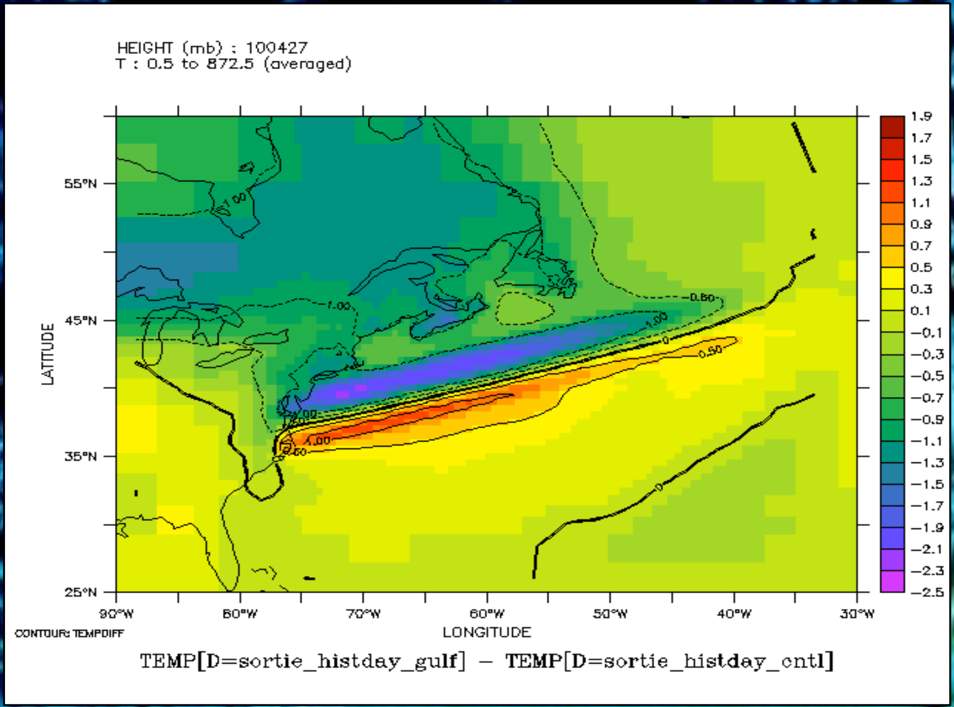
IPCC-class GCM: LMD-Z

Climatology



Air temperature (K)

Superimpose *f* (*T*)= 2cos(*x*)*(–8)sin(*y*), for a Gulf Stream that has an axis inclination of 25° to zonal.

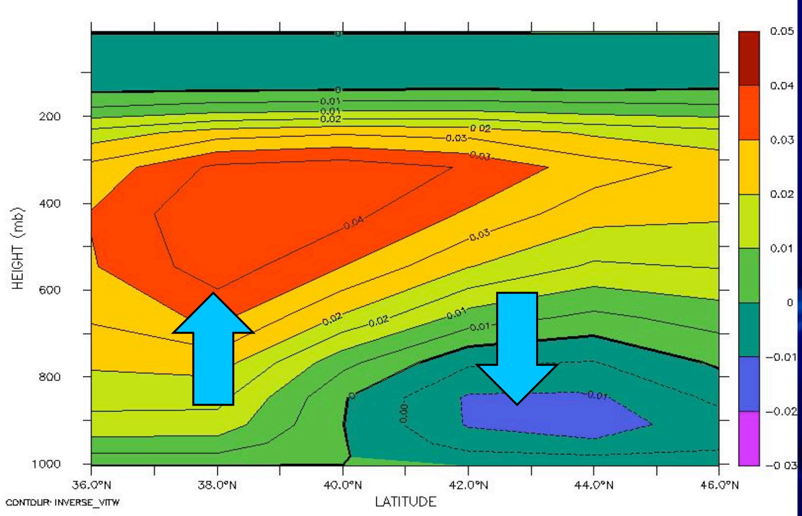


TEMP[D=sortie_histday_gulf] − TEMP[D=sortie_histday_cntl]

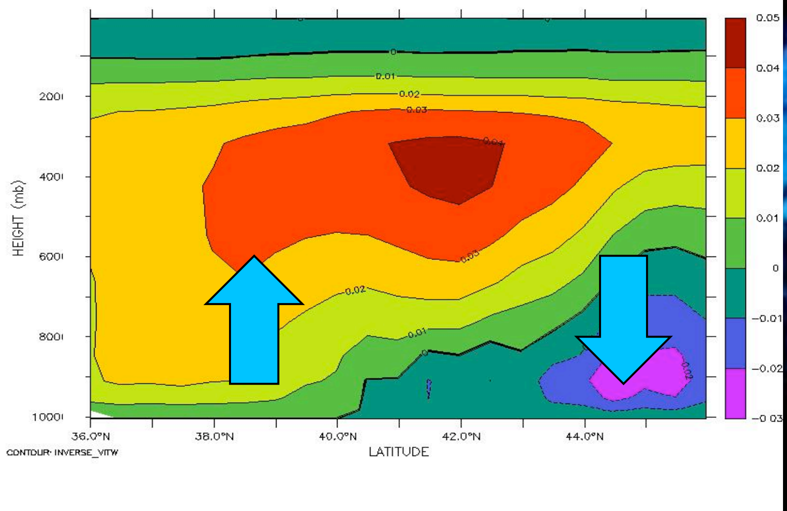
Results

Mean *w* averaged from 70°W to 40°W.

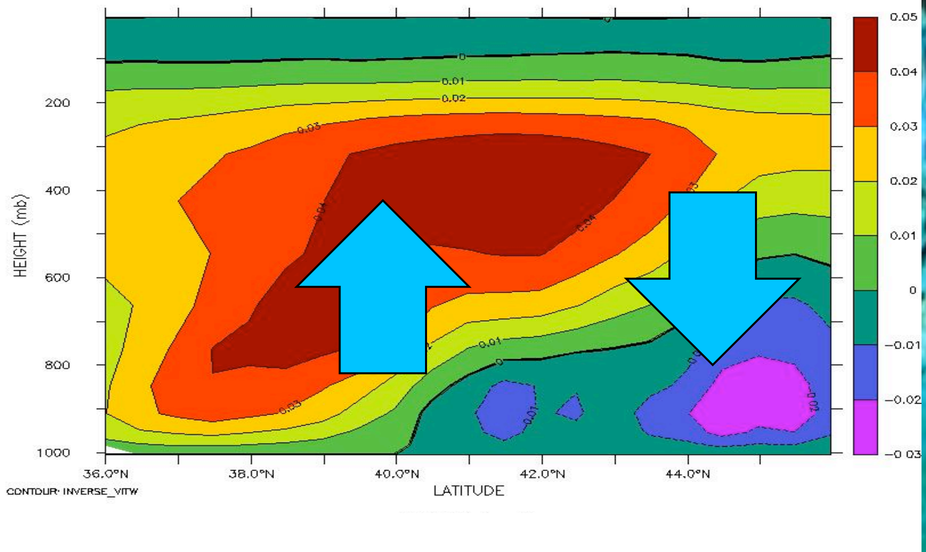
Height *vs*. latitude cross-section; red/blue means +ve/–ve upward velocity.



Without Zoom & without SST front



With Zoom & without SST front



With Zoom & with SST front

Can we, nonlinear people, help?

The uncertainties might be ***intrinsic***, rather than mere "tuning problems"

If so, maybe ***stochastic structural stability*** *could help*!

Might fit in nicely with recent taste for "stochastic parameterizations"

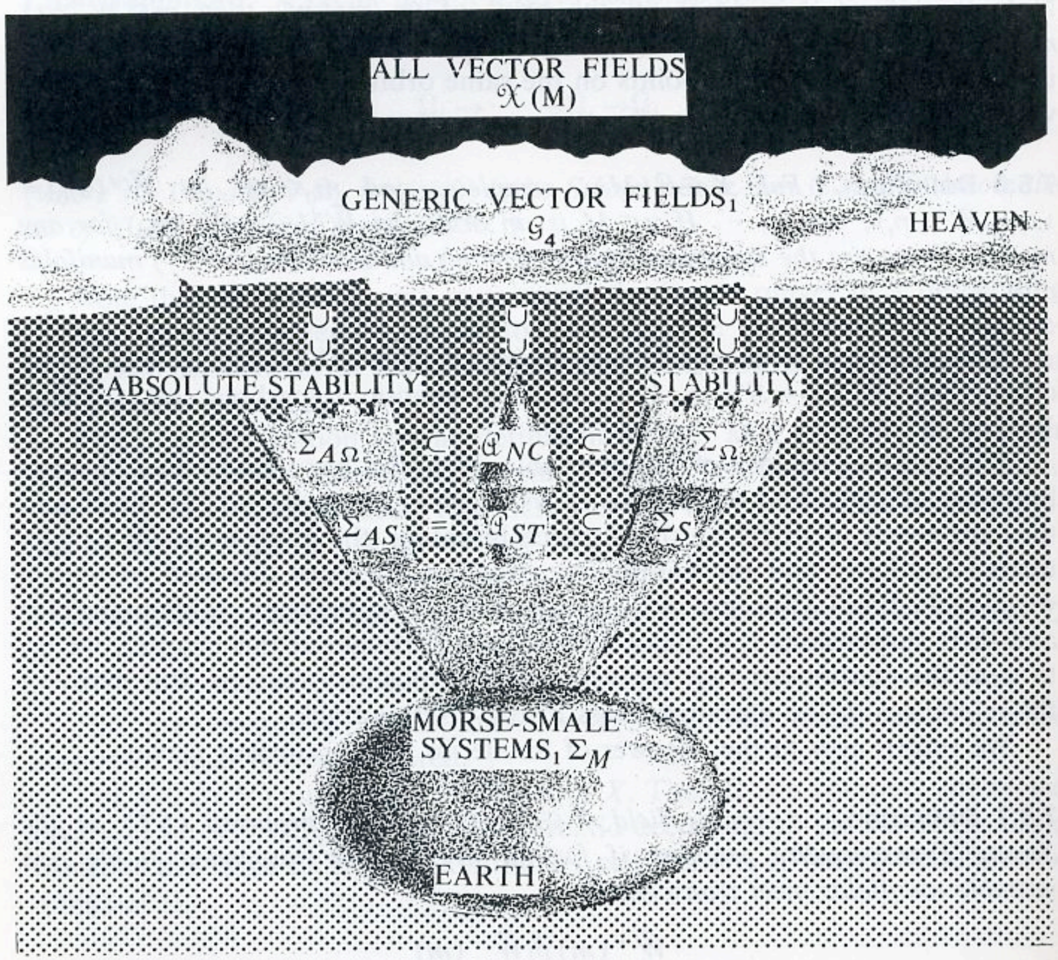
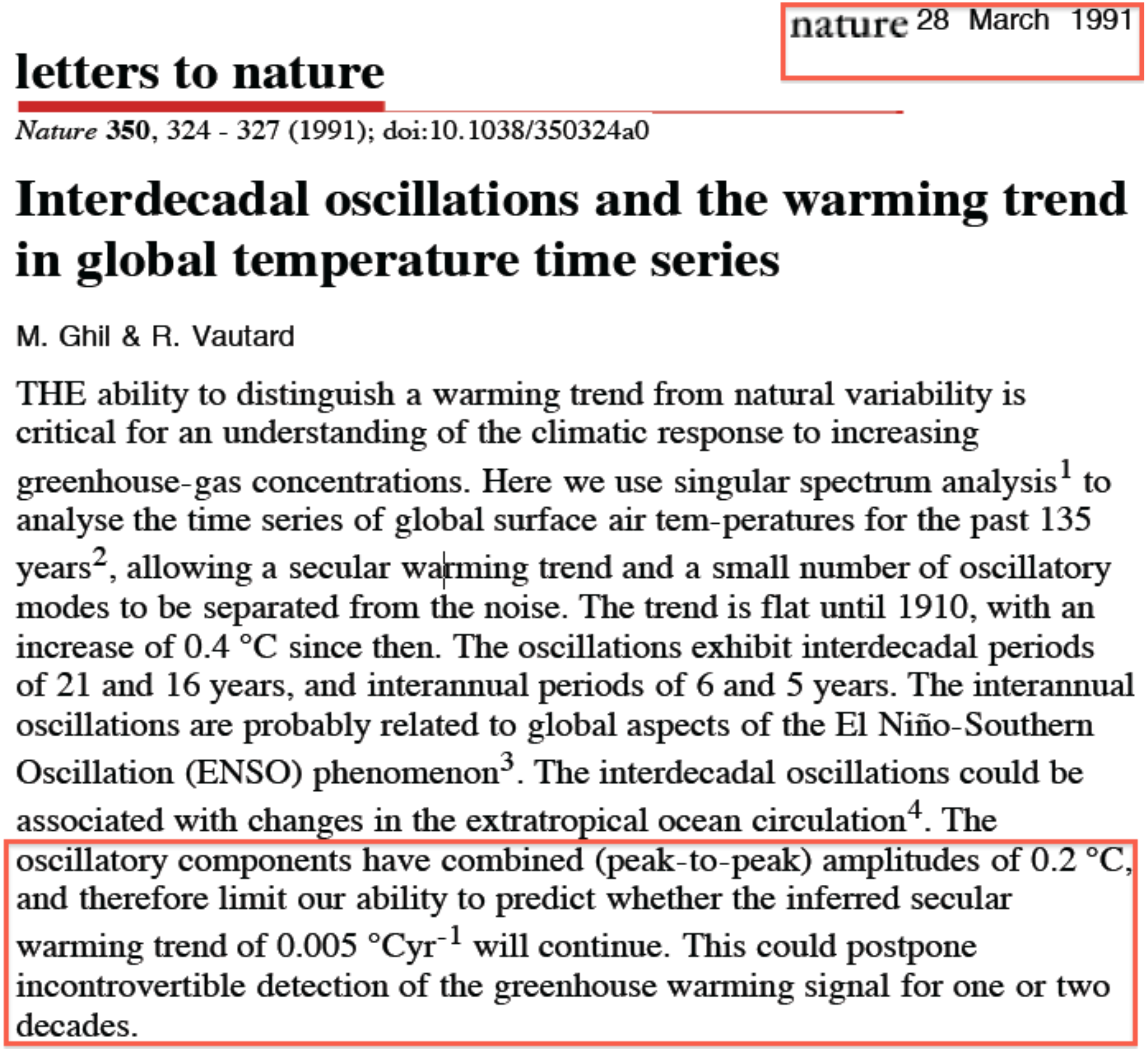


Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from *Abraham & Marsden*, 1978)

A little more on natural variability, I



nature 28 March 1991

letters to nature

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Interdecadal oscillations and the warming trend in global temperature time series

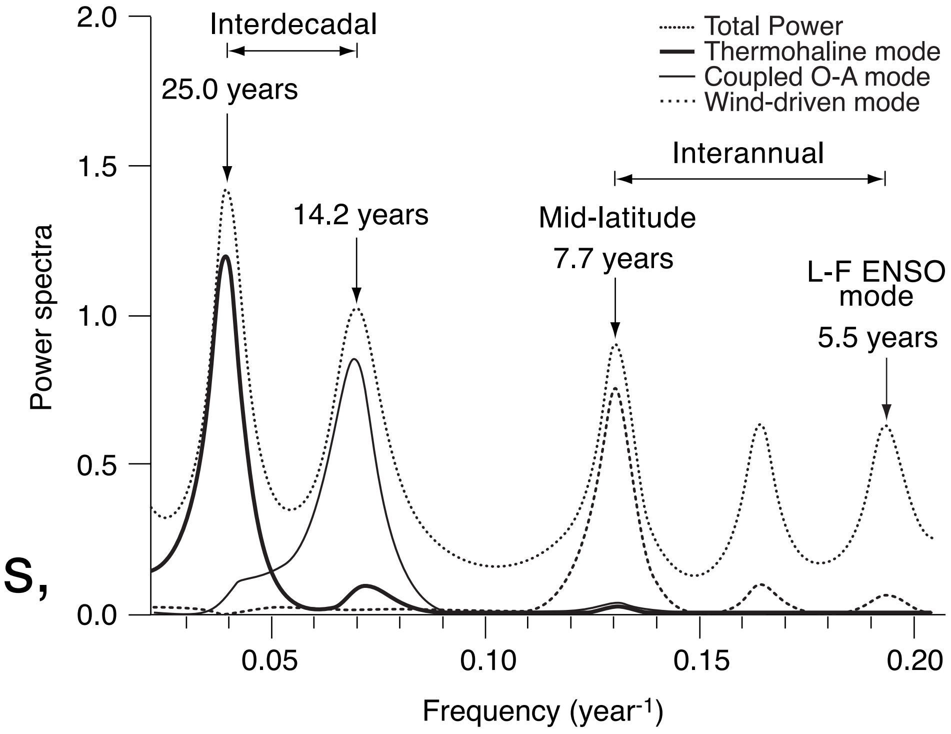
M. Ghil & R. Vautard

THE ability to distinguish a warming trend from natural variability is critical for an understanding of the climatic response to increasing greenhouse-gas concentrations. Here we use singular spectrum analysis[1] to analyse the time series of global surface air tem-peratures for the past 135 years[2], allowing a secular warming trend and a small number of oscillatory modes to be separated from the noise. The trend is flat until 1910, with an increase of 0.4 °C since then. The oscillations exhibit interdecadal periods of 21 and 16 years, and interannual periods of 6 and 5 years. The interannual oscillations are probably related to global aspects of the El Niño-Southern Oscillation (ENSO) phenomenon[3]. The interdecadal oscillations could be associated with changes in the extratropical ocean circulation[4]. The oscillatory components have combined (peak-to-peak) amplitudes of 0.2 °C, and therefore limit our ability to predict whether the inferred secular warming trend of 0.005 °Cyr^{-1} will continue. This could postpone incontrovertible detection of the greenhouse warming signal for one or two decades.

A little more on natural variability, II

There are several natural modes of variability, internal to the climate system:
El Niño/Southern Oscillation (*ENSO*),
North Atlantic Oscillation (*NAO*),
Pacific Decadal Oscillation (*PDO*), etc.

It is the chaotic interaction of these modes that is forced by us, not some dumb equilibrium.



Plaut, Ghil & Vautard (*Science*, 1995)