Lezioni dottorali, Università degli Studi di Napoli "Parthenope"

Napoli, 21–27 October 2016

Nonlinearity, Tipping Points & Chaos in the Climate Sciences

Michael Ghil

Ecole Normale Supérieure, Paris, and
University of California, Los Angeles

Please visit these sites for more info.

https://dept.atmos.ucla.edu/tcd
http://www.environnement.ens.fr/

Overall Outline

- **Lecture I: EBMs(+), paleoclimate & "tipping points"**
- **Lecture II: The wind-driven ocean circulation**
- **Lecture III: Advanced spectral methods–SSA(±) *et al*.**

➔**Lecture IV: Weather & climate—**
deterministic or stochastic?

(+) EBM = Energy balance model
(±) SSA = Singular-spectrum analysis
(❖) RDS = Random dynamical system

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Weather and Climate: Deterministic or Stochastic?

Michael Ghil

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Joint work with many people over the years

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A little history of weather and chaos – I

- The 19th century made substantial advances in grasping *meteorological phenomena* in mid-latitudes, via land and sea observations.
- At the same time, the physical formulation and the mathematical analysis of the *partial differential equations (PDEs)* governing fluid flows (Navier-Stokes, shallow-water) made great strides.
- V. Bjerknes (1904) and L.F. Richardson (1922) were the first to formulate and solve the problem of *weather prediction as an initial-value problem* for a systeme of PDEs; v. also F. Exner (1904) and M. Margules (1904).
- With the *spectacular growth of the number of meteorological observations* during and after WWII, routine *numerical weather prediction (NWP)* will start in the 1960s.

A little history of weather and chaos – II

- During the 1940s and 1950s, J. von Neumann assembles around him a group of meteorologists for the first *experimental weather forecasts* (Charney, Fjørtoft & V. Neumann, 1950).
- P.D. Thompson (1961) organizes the transition from these *experimental forecasts* at Princeton (Inst. Adv. Studies) to *routine, operational NWP* via the JNWPU in Washington, DC.
- *Operational NWP* gets better & better, due to improvements in the observations (satellites & other obs. systems), of the PDE models being used (subgrid-scale "parametrizations," etc.), of the numerical methods used to solve them, of the data assimilation methods applied to combine the data and the models, and so on. But, oh heck ($✪❖⌘!): *these routine forecasts are* (still) *far from perfect?!?!*
- At that point, E.N. Lorenz (1963) formulates the problem of loss of forecast skill in NWP as one of *stability and predictability of nonlinear dynamical systems.*

Outline

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  - a little history of deterministic chaos
  - the Lorenz convection model & the butterfly effect
  - sensitivity to initial data ➔ local error growth
- Climate & randomness
  - the IPCC process: results and uncertainties
  - sensitivity to model formulation ➔ errors in the statistics
- Uncertainties and how to fix them
  - structural stability and other kinds of robustness
  - time-dependent forcing & what to do with it
- Two illustrative examples
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The sources of deterministic chaos – a bit of bibliometry (*)

- ***Mathematics*** :

  Poincaré (1890), Hadamard (1898), Smale (1968), etc.

  Poincaré (1889, book) = 223, (1890, article) = 50 ;

  Smale (1968) = 1423

  v. also Van der Pol, Cartwright & Littlewood, etc.
- ***Physics*** :

  D. Ruelle and F. Takens, 1971: On the nature of turbulence. *Commun. Math. Phys.*, **20**, 167–192 = 1318.
- ***Meteorology and prediction***:

  Lorenz (1963a) = 4832.

(*) Just for laughs, of course, according to
Thomson Reuters Web of Knowledge,
except for Poincaré (Google Scholar) – 6 May 2009.

The great surprise of deterministic chaos – a major discovery of the 20th century?

- ***The "classical" point of view***

  The kinetic theory of gases (L. Boltzmann), statistical mecanics (J.W. Gibbs), Einstein's explanation (1905, *annus mirabilis*) of Brownian motion led one to believe that irregular behavior in a medium could only result from the interaction of an infinite (or at least very large) number of particles.

- ***The surprise of the '60s and '70s***

  Three ordinary differential equations (ODEs) were enough to generate irregular behavior!!

- ***Why 3 and not just 2 variables?***

  In the "phase space" of a flow in one dimension (1-D) there can be only *fixed points* (FPs, i.e. equilibria);
  in 2-D only FPs and *limit cycles* (LCs, i.e. periodic solutions)(*);
  one needs 3-D to "accommodate" a "*strange attractor*"!

(*) The Jordan curve theorem and the Poincaré-Bendixson theorem explain this: no self-intersections!

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The Lorenz model (1963a): a concrete example of a strange attractor(*)

- ***The model equations: 3 coupled, nonlinear ODEs***

$$\begin{aligned} \dot{X} &= & & -\sigma X & & +\sigma Y \qquad (1) \\ \dot{Y} &= & -XZ & +rX & & -Y \qquad (2) \\ \dot{Z} &= & XY & -bZ & & \qquad (3) \end{aligned}$$

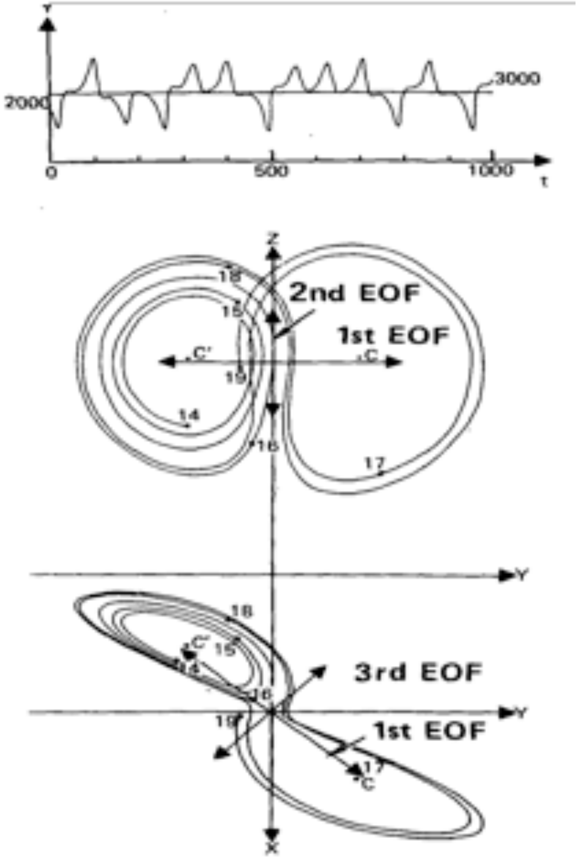
- ***Physics: a model of thermal convection in 2-D***

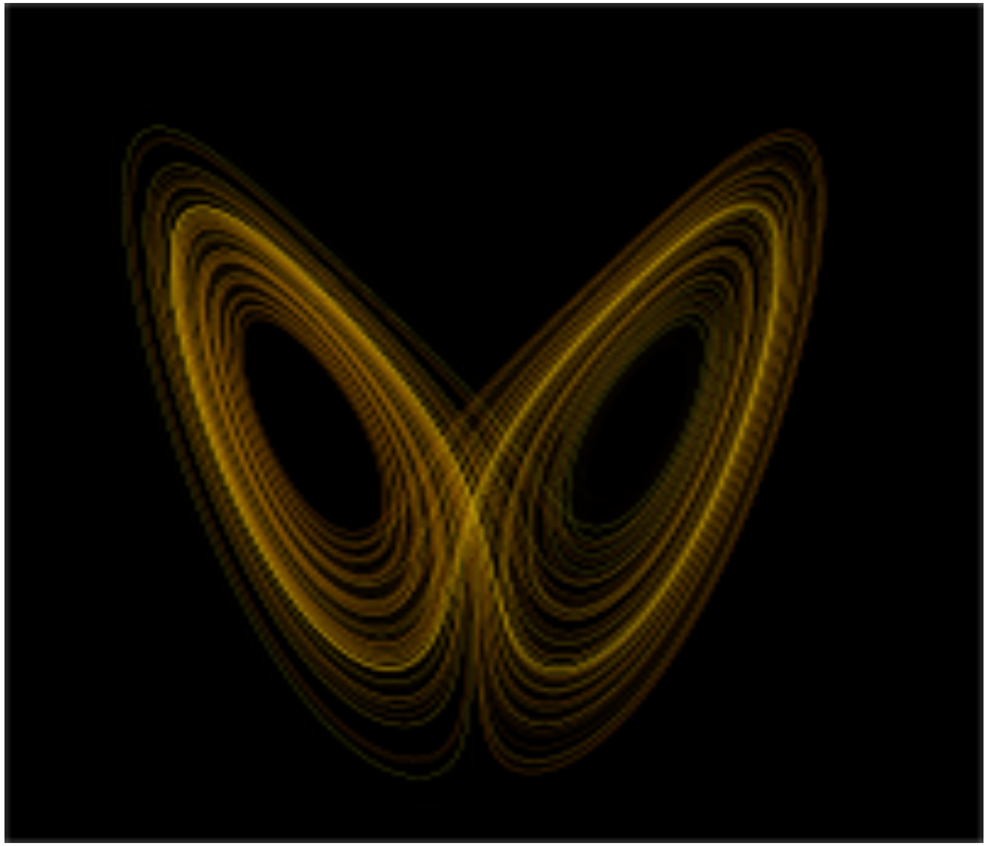
The variables *X* and *Y* represent the intensity of the velocity field in a 2-D space, *Z* is the deviation of the vertical temperature profile from pure conduction (no motion), and $(X, Y, Z)^{\bullet}$ is their rate of change.

The parameters are the Rayleigh number *ρ* (intensity of the thermal forcing), the Prandtl number σ (the fluid's dissipative properties) and *β* caracterizes the wave length of the perturbation from pure conduction.

(*) Mommy, what's a strange attractor, please?

The Lorenz model (1963a)
– some numerical solutions





Plot of $Y = Y(t)$ + projections onto the (X, Y) & (Y, Z) planes

Trajectory in phase space

Both for the canonical "chaotic" values $\rho = 28$, $\sigma = 10$, $\beta = 8/3$.

Edward Norton Lorenz
May 23, 1917 – April 16, 2008

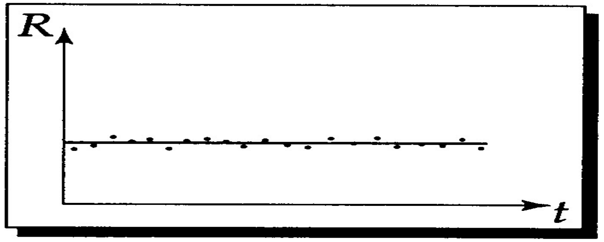
Jule Gregory Charney
January 1, 1917 – June 16, 1981

Prediction and Predictability

1. Easiest to predict:

constant phenomena

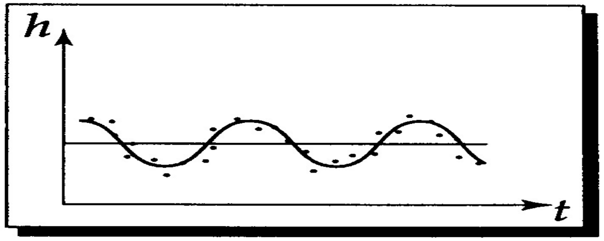
e.g., the radius of the Earth *R* –
only need 1 number



2. A little harder:

periodic phenomena

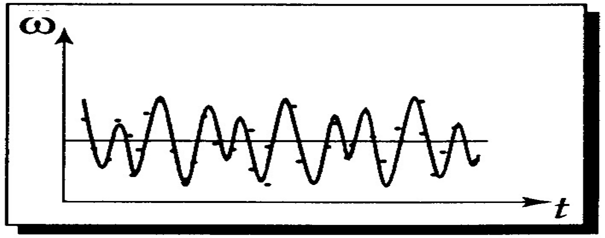
e.g., sunrise, tides – only need 3 numbers :
period, amplitude & phase.



3. Even harder:

multi-periodic phenomena

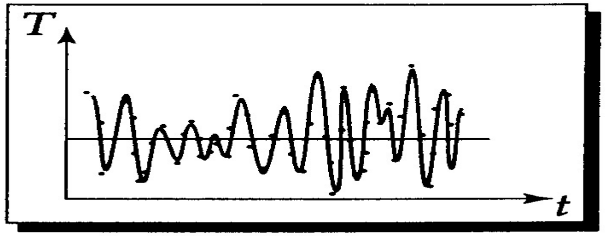
e.g., celestial mecanics –
need (finitely) many numbers



4. Hardest:

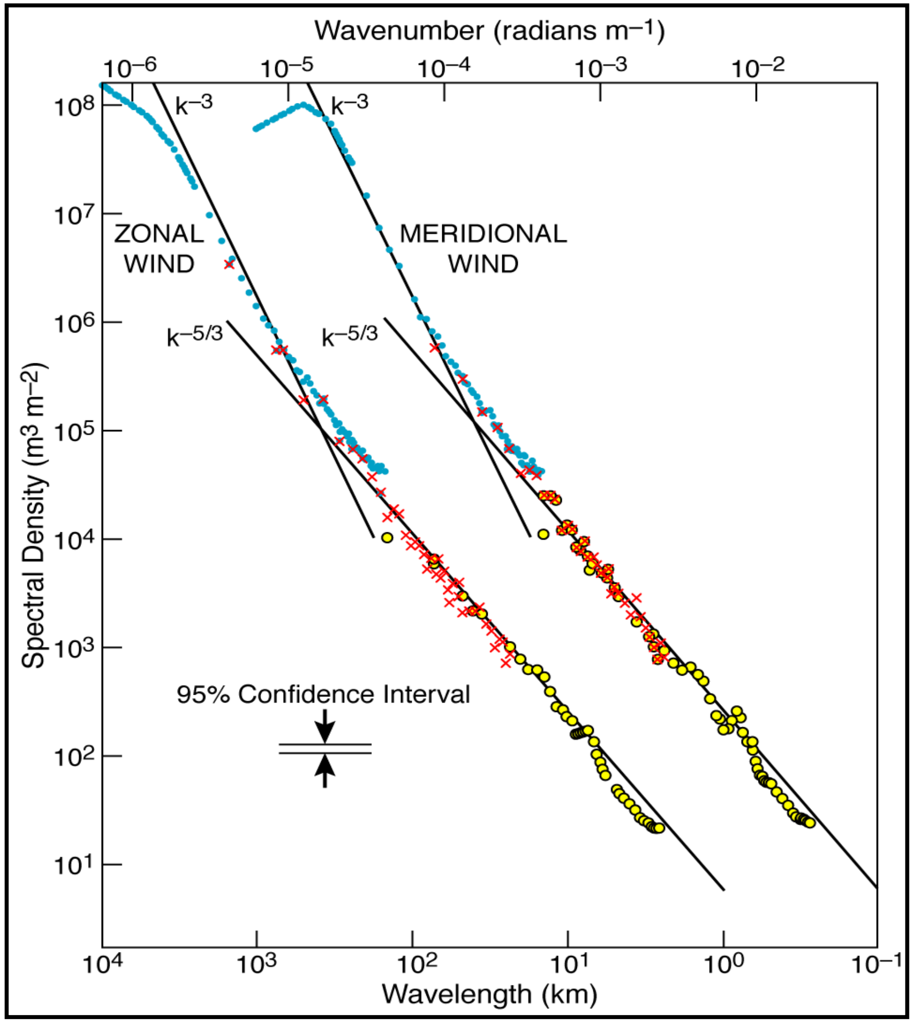
aperiodic phenomena

e.g., thermal convection, weather –
infinitely many numbers



But deterministic chaos doesn't explain all: there are many other sources of irregularity!

- The energy spectrum of the atmosphere and ocean is "full": all space & time scales are active and they all contribute to forecasting uncertainties.
- Still, one can imagine that the longest & slowest scales contribute most to the longest-term forecasts.
- "One person's signal is another person's noise."

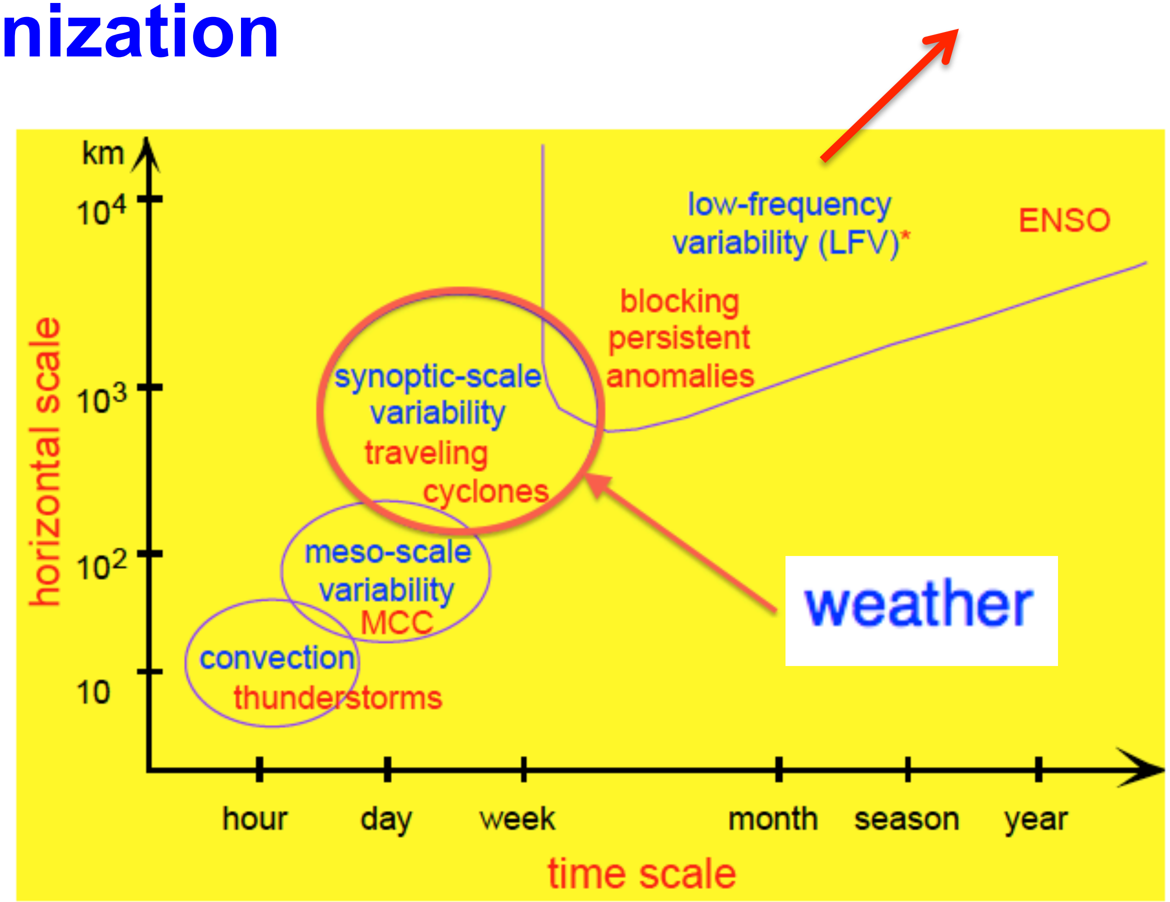


After Nastrom & Gage (*JAS*, 1985)

Multiple scales of motion: Space-time organization

Observations

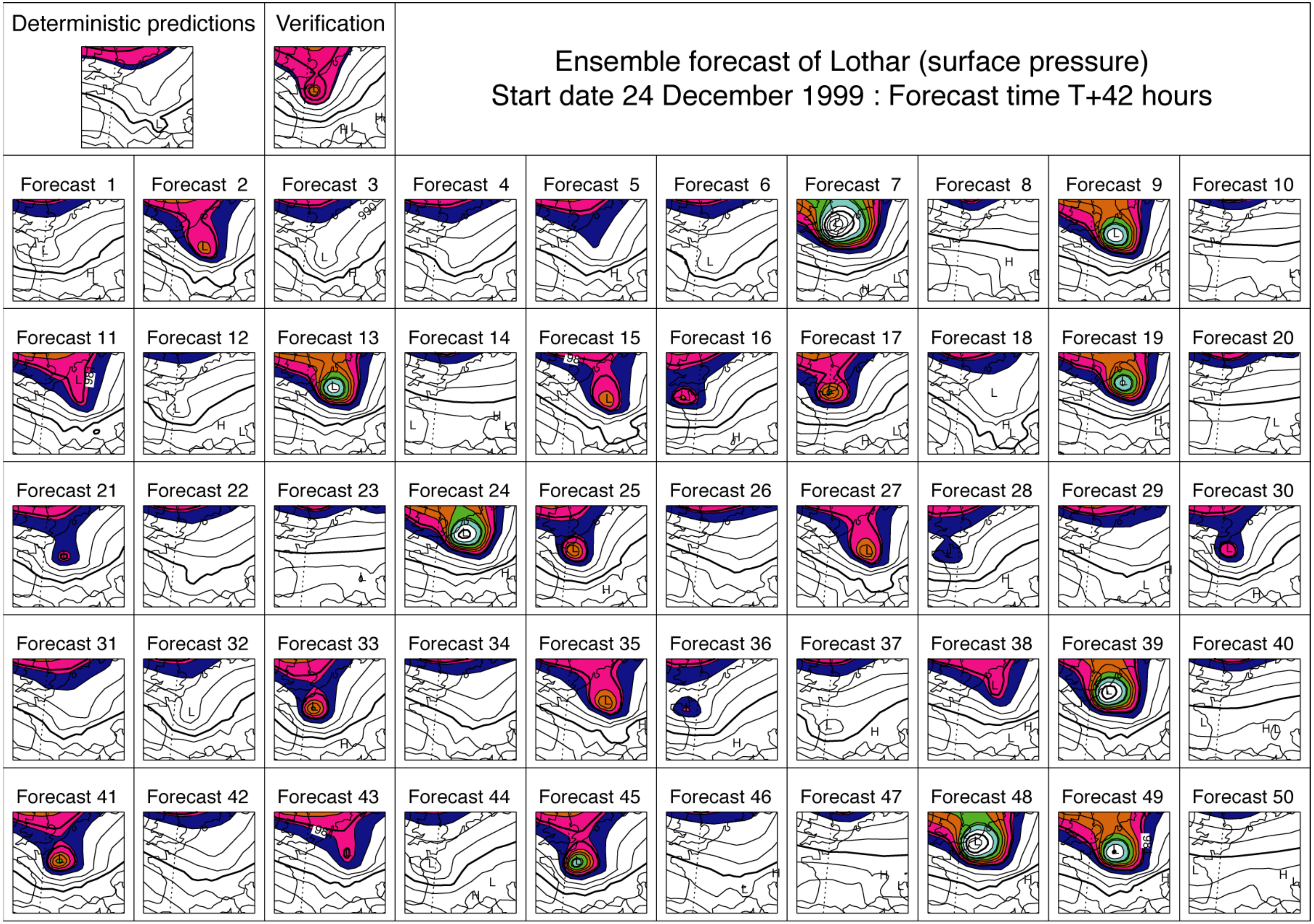
- The most active scales lie along a diagonal in this space *vs.* time plot.
- Why this is so is far from clear as of now.
- We'll deal with weather first, then climate.



N.B. A **high-variability ridge** lies close to the **diagonal** of the plot (*cf.* also Fraedrich & Böttger, 1978, *JAS*)

* **LFV** ≅ 10–100 days (intraseasonal)

Ensemble forecast of Lothar (surface pressure)
Start date 24 December 1999 : Forecast time T+42 hours



Courtesy Tim Palmer, 2009

Exponential divergence vs. "coarse graining"

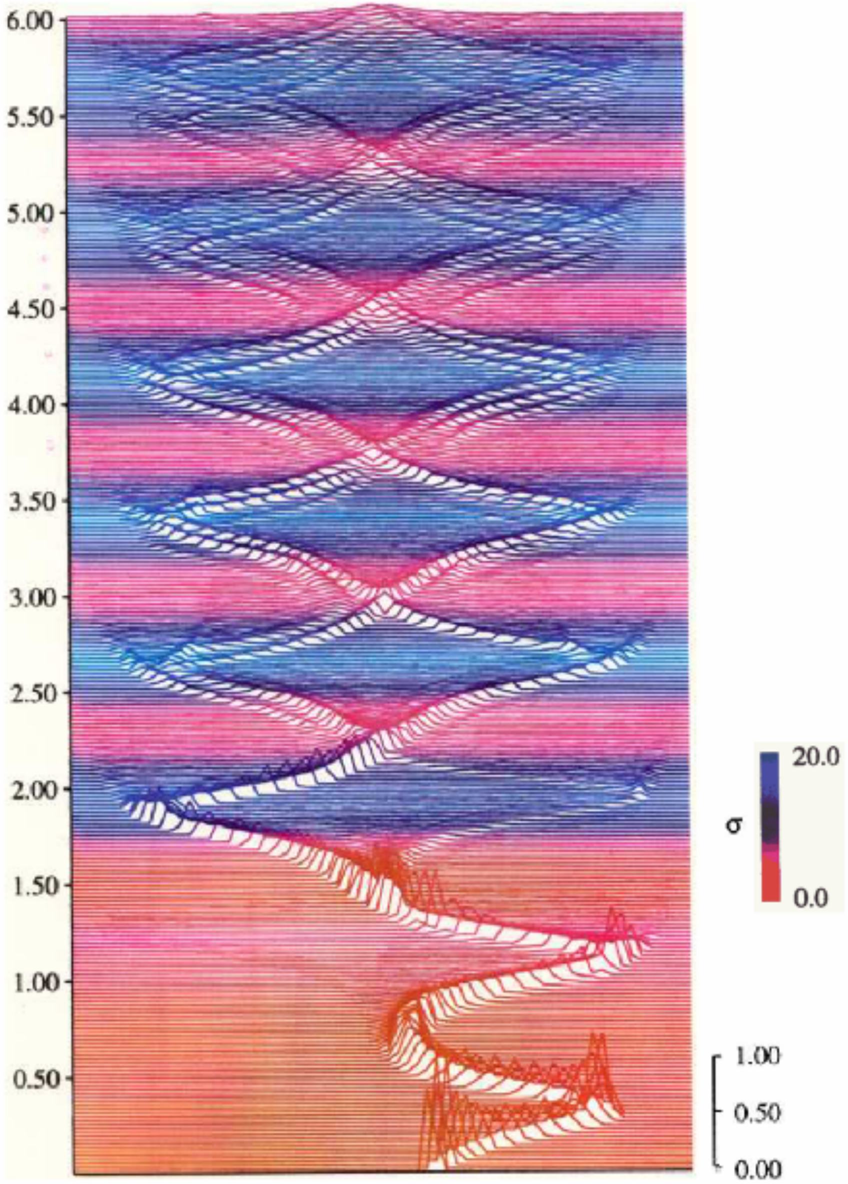
The classical view of dynamical systems theory is:

positive Lyapunov exponent ➔ trajectories diverge exponentially

But the presence of multiple regimes implies a much more structured behavior in phase space

Still, the probability distribution function (pdf), when calculated forward in time, is pretty smeared out

L. A. Smith (*Encycl. Atmos. Sci.*, 2003)



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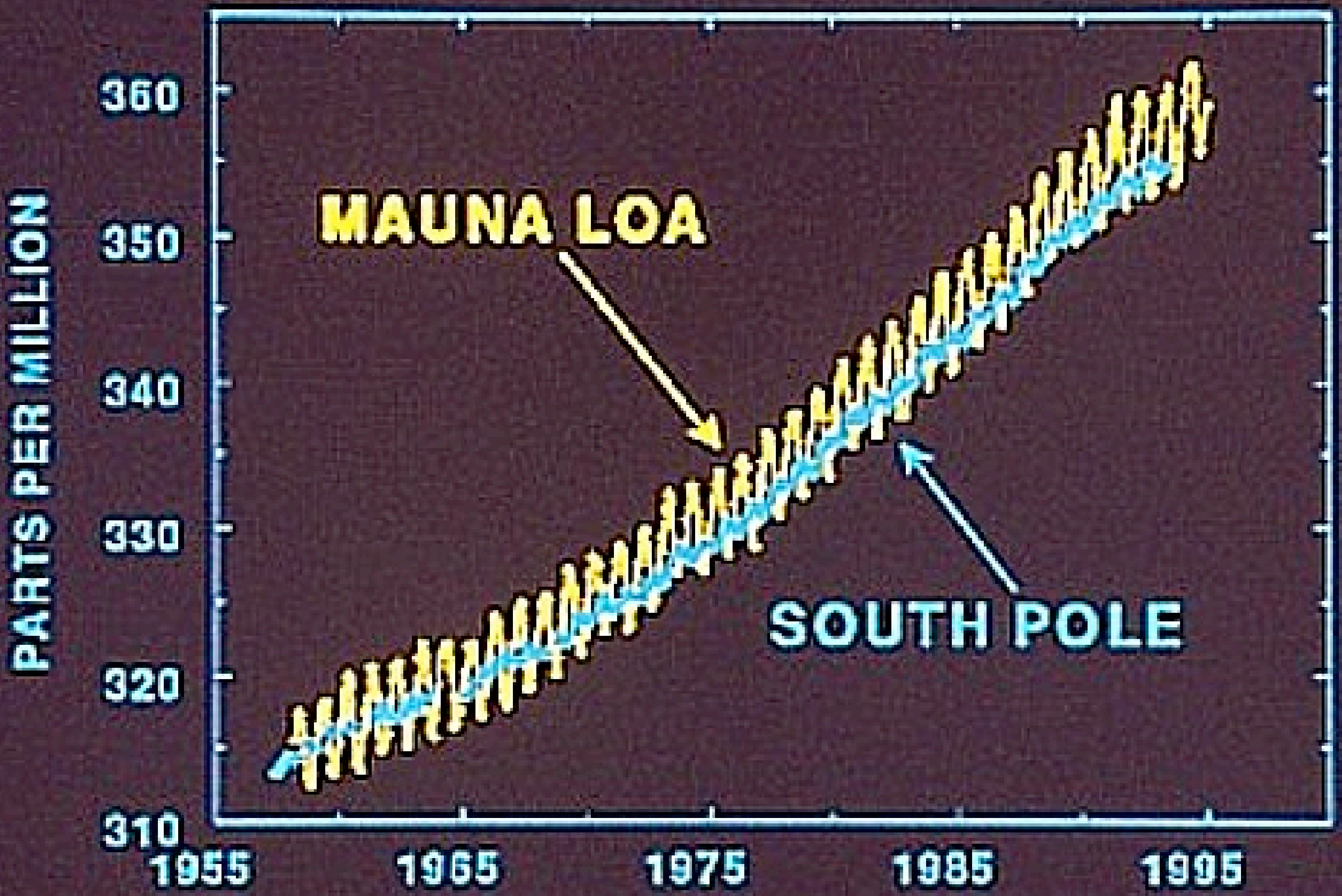
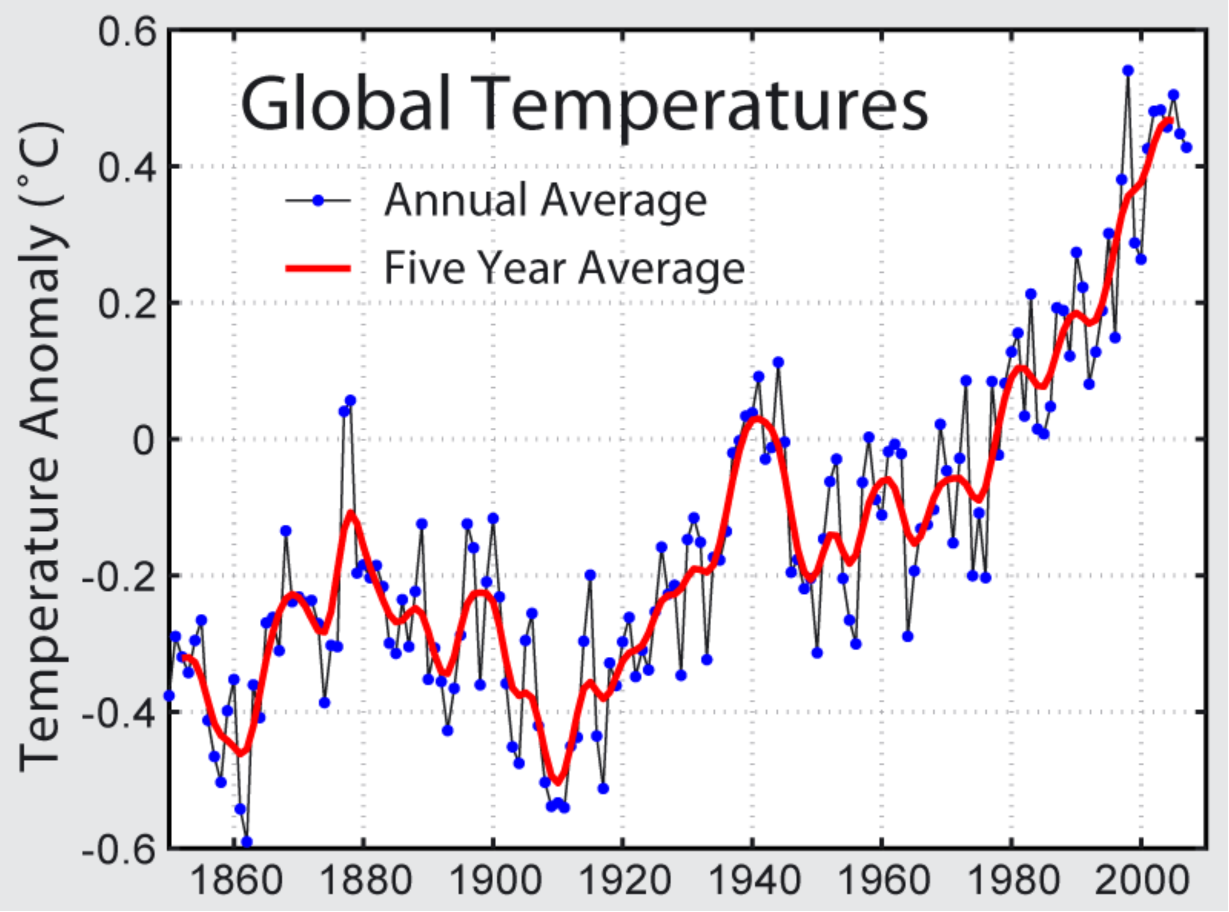
CO2 IN THE ATMOSPHERE



Temperatures and GHGs

Greenhouse gases (GHGs) go up,
temperatures go up:

It’s gotta do with us, at least a bit,
doesn’t it?



Wikicommons, from
Hansen *et al.* (*PNAS*, 2006);
see also http://data.giss.nasa.gov/gistemp/graphs/

Unfortunately, things aren’t all that easy!

What to do?

Try to achieve better interpretation of, and agreement between, models …

Ghil, M., 2002: Natural climate variability, in *Encyclopedia of Global Environmental Change*, T. Munn (Ed.), Vol. 1, Wiley

Natural variability introduces additional complexity into the anthropogenic climate change problem

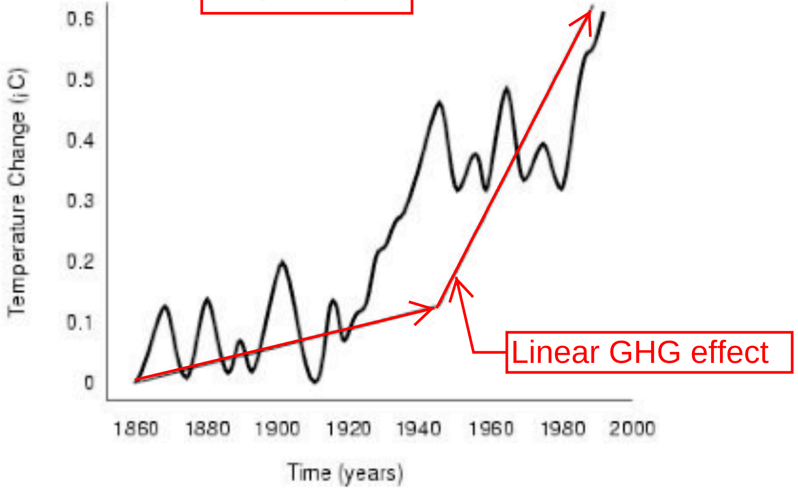
The most common interpretation of observations and GCM simulations of climate change is still in terms of a scalar, linear Ordinary Differential Equation (ODE)

$$\boxed{c\frac{dT}{dt} = -kT + Q}$$

$$\boxed{\begin{aligned} k &= \sum k_i \\ Q &= \sum Q_j \\ Q_j &= Q_j(t) \end{aligned}}$$

– feedbacks (+ve and -ve)

– sources & sinks



Linear response to CO_2 vs. observed change in T

Hence, we need to consider instead a system of nonlinear Partial Differential Equations (PDEs), with parameters and multiplicative, as well as additive forcing (deterministic + stochastic)

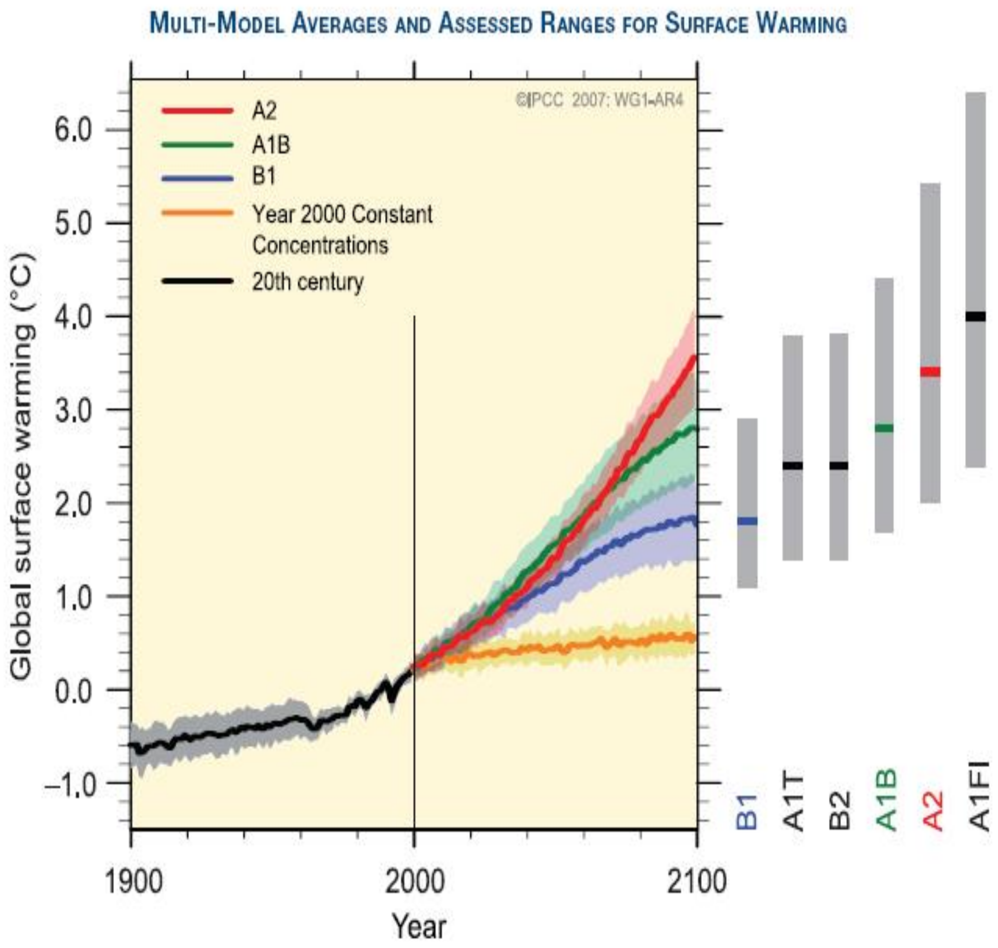
$$\boxed{\frac{d\boldsymbol{X}}{dt} = \boldsymbol{N}(\boldsymbol{X}, t, \mu, \beta)}$$

Global warming and its socio-economic impacts – I

Temperatures rise:

- What about impacts?
- How to adapt?

The answer, my friend, is blowing in the wind, *i.e.*, it depends on the accuracy and reliability of the forecast …



Multi-Model Averages and Assessed Ranges for Surface Warming

Figure SPM.5. *Solid lines are multi-model global averages of surface warming (relative to 1980–1999) for the scenarios A2, A1B and B1, shown as continuations of the 20th century simulations. Shading denotes the ±1 standard deviation range of individual model annual averages. The orange line is for the experiment where concentrations were held constant at year 2000 values. The grey bars at right indicate the best estimate (solid line within each bar) and the* ***likely*** *range assessed for the six SRES marker scenarios. The assessment of the best estimate and* ***likely*** *ranges in the grey bars includes the AOGCMs in the left part of the figure, as well as results from a hierarchy of independent models and observational constraints. {Figures 10.4 and 10.29}*

Source : IPCC (2007),
AR4, WGI, SPM

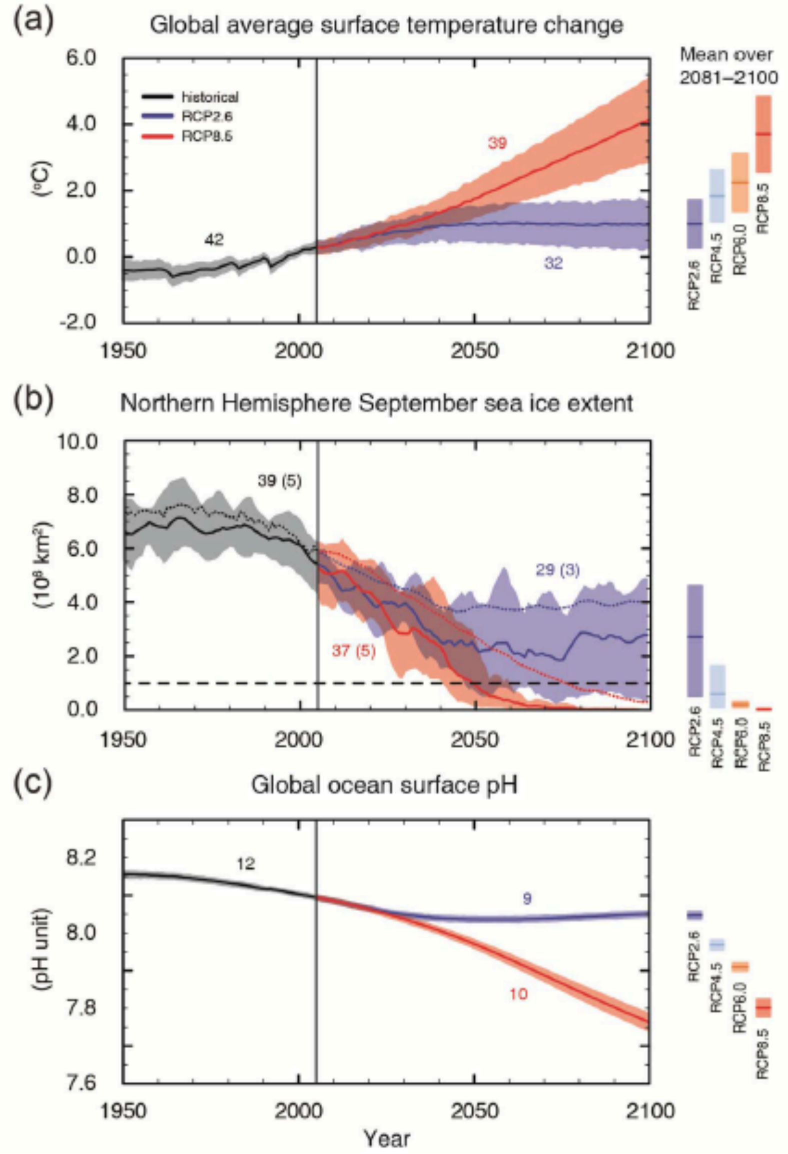
Global warming and
its socio-economic impacts– II

Temperatures rise:

- What about impacts?
- How to adapt?

AR5 vs. AR4

A certain air of *déjà vu*:
GHG "scenarios" have been replaced by "representative concentration pathways" (RCPs), more dire predictions, but the uncertainties remain.



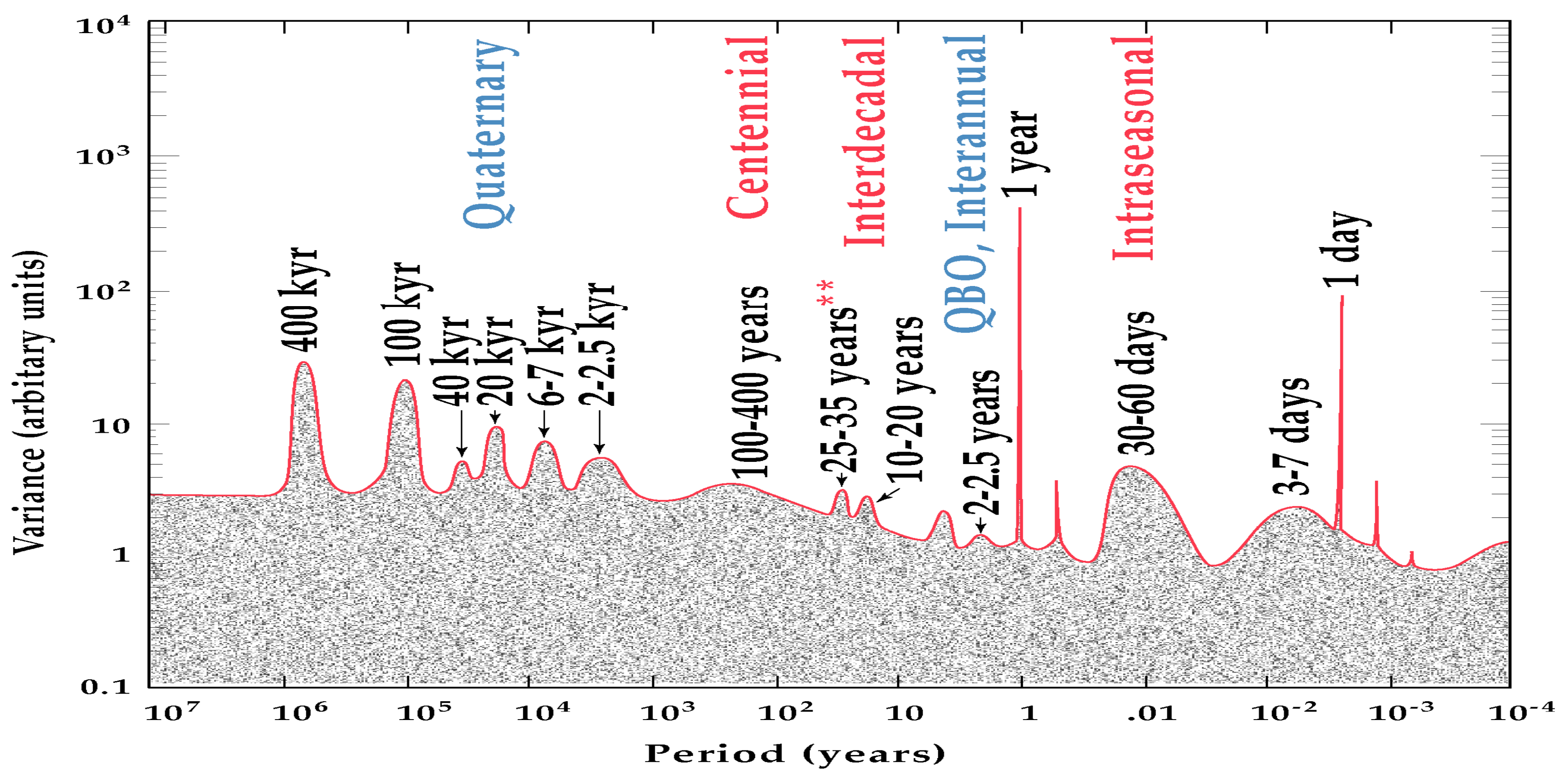
Source : IPCC (2013), AR5, WGI, SPM

Composite spectrum of climate variability

Standard treatement of frequency bands:

1. High frequencies – white noise (or "colored")

2. Low frequencies – slow evolution of parameters



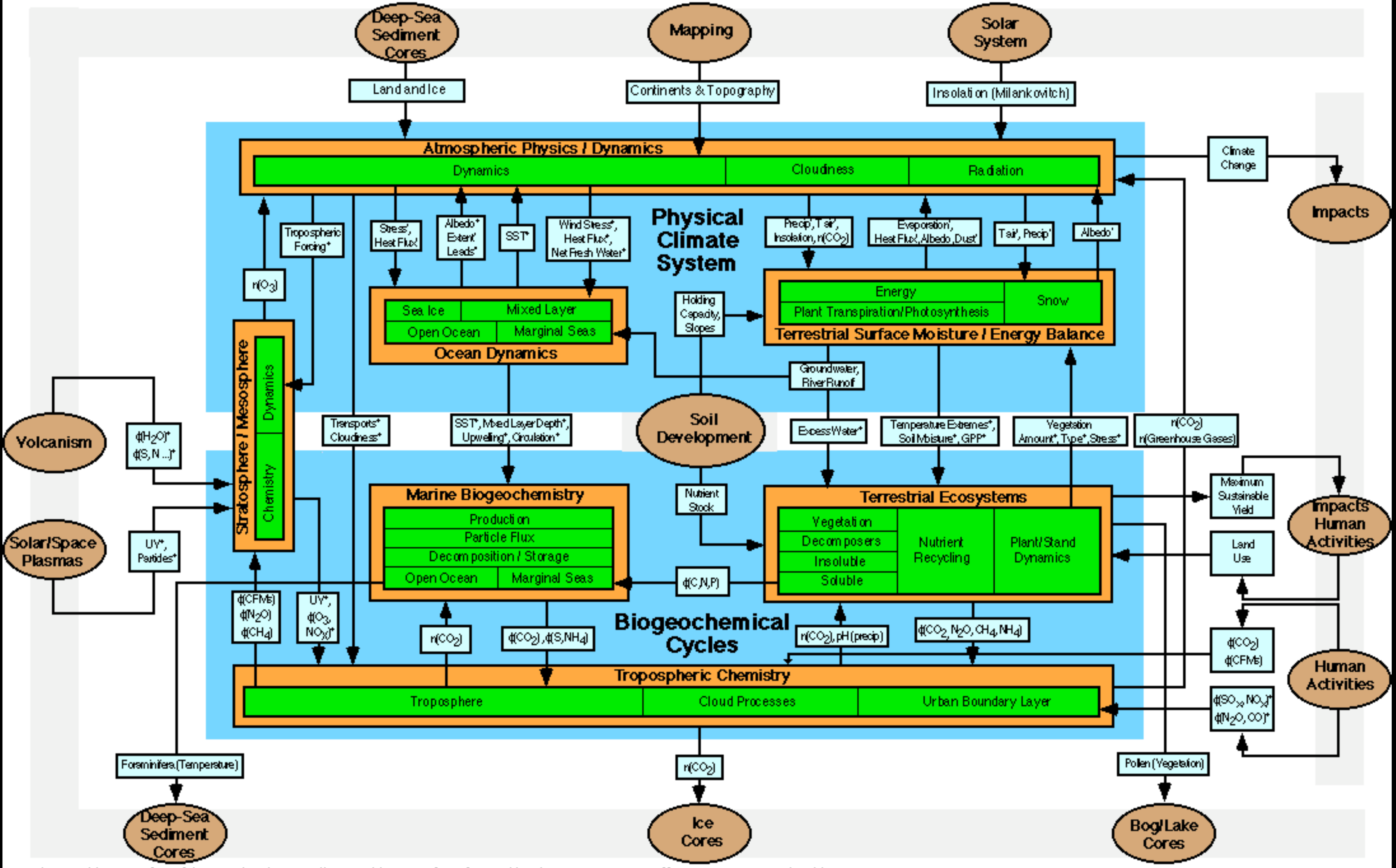
From **Ghil (2001, EGEC)**, after **Mitchell*** **(1976)**

*** "No known source of deterministic internal variability"**

**** 27 years – Brier (1968, *Rev. Geophys.*)**

•F. Bretherton's "horrendogram" of Earth System Science

CONCEPTUAL MODEL of Earth System process operating on timescales of decades to centuries



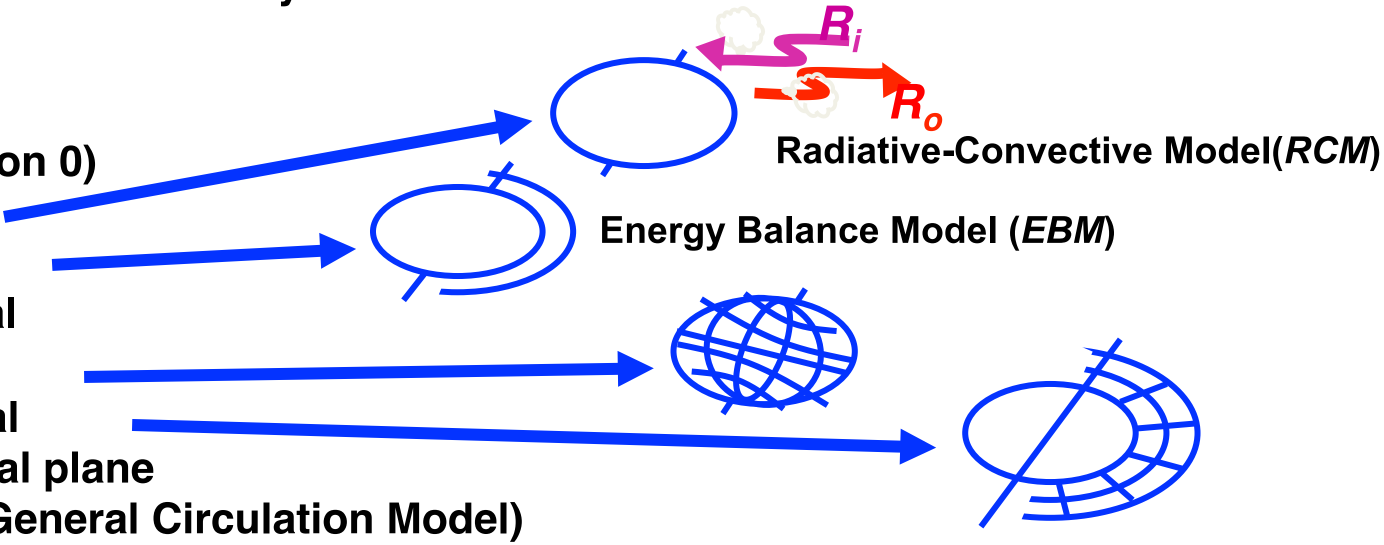
' = on timescale of hours to days   * = on timescale of months to seasons ϕ = flux   n = concentration

Earth System Science Overview, *NASA Advisory Council*, 1986

Climate models (atmospheric & coupled) : A classification

- *Temporal*
  - stationary, (quasi-)equilibrium
  - transient, climate variability

- *Space*
  - 0-D (dimension 0)
  - 1-D
    - vertical
    - latitudinal
  - 2-D
    - horizontal
    - meridional plane
  - 3-D, GCMs (General Circulation Model)
  - Simple and intermediate 2-D & 3-D models



Radiative-Convective Model(*RCM*)

Energy Balance Model (*EBM*)

- *Coupling*
  - Partial
    - unidirectional
    - asynchronous, hybrid
  - Full

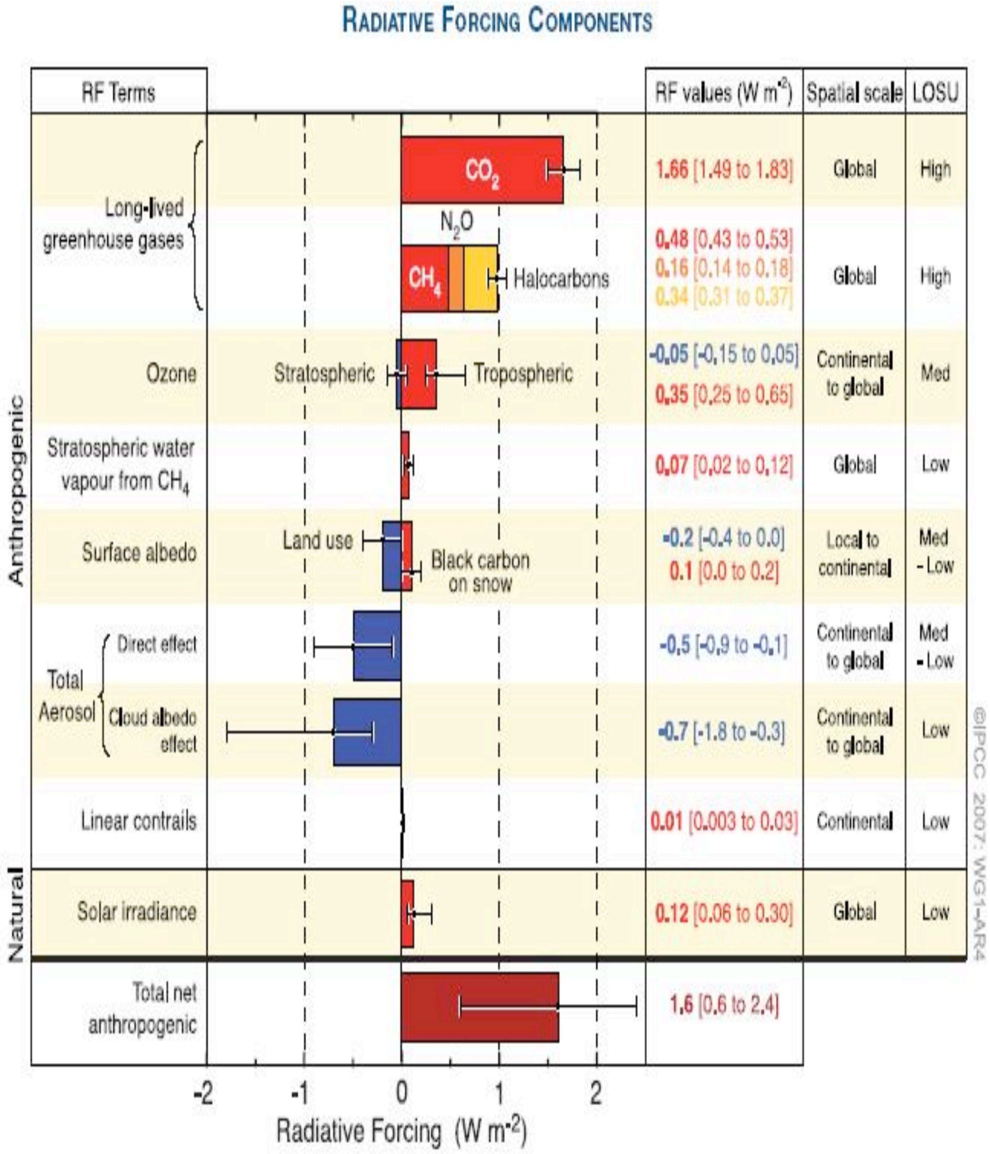
➔ ***Hierarchy*:** back-and-forth between the simplest and the most elaborate model, and between the models and the observational data

GHGs rise!

It’s gotta do with us, at least a bit, ain’t it?

But just how much?

IPCC (2007)



So what's it gonna be like, by 2100?

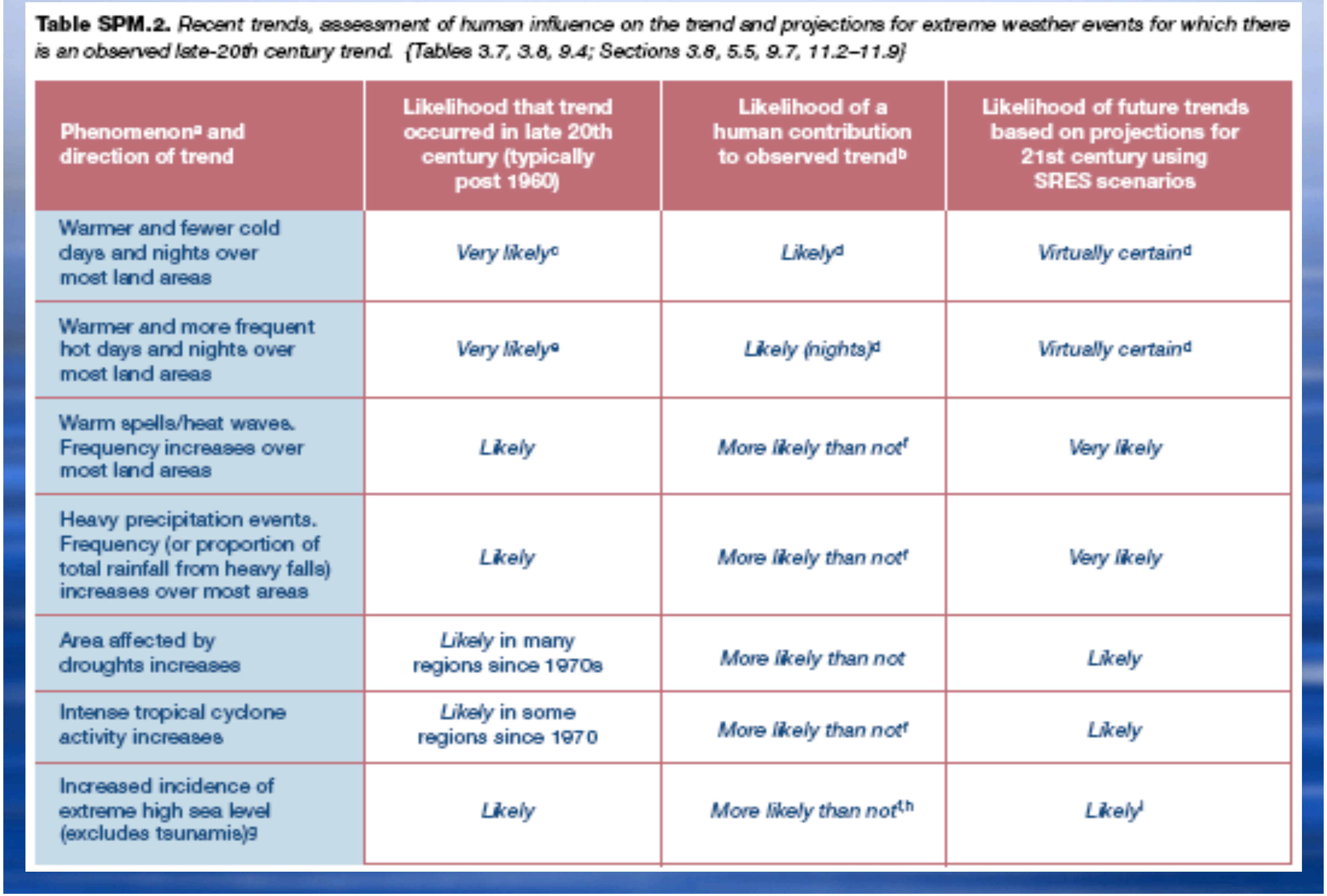


Table SPM.2. *Recent trends, assessment of human influence on the trend and projections for extreme weather events for which there is an observed late-20th century trend.* {Tables 3.7, 3.8, 9.4; Sections 3.8, 5.5, 9.7, 11.2–11.9}

Phenomenon[a] and direction of trend	Likelihood that trend occurred in late 20th century (typically post 1960)	Likelihood of a human contribution to observed trend[b]	Likelihood of future trends based on projections for 21st century using SRES scenarios
Warmer and fewer cold days and nights over most land areas	*Very likely*[c]	*Likely*[d]	*Virtually certain*[d]
Warmer and more frequent hot days and nights over most land areas	*Very likely*[e]	*Likely (nights)*[d]	*Virtually certain*[d]
Warm spells/heat waves. Frequency increases over most land areas	*Likely*	*More likely than not*[f]	*Very likely*
Heavy precipitation events. Frequency (or proportion of total rainfall from heavy falls) increases over most areas	*Likely*	*More likely than not*[f]	*Very likely*
Area affected by droughts increases	*Likely* in many regions since 1970s	*More likely than not*	*Likely*
Intense tropical cyclone activity increases	*Likely* in some regions since 1970	*More likely than not*[f]	*Likely*
Increased incidence of extreme high sea level (excludes tsunamis)[g]	*Likely*	*More likely than not*[f,h]	*Likely*[i]

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Can dynamical systems theory help?

The uncertainties might be ***intrinsic***, rather than mere "tuning problems"

If so, maybe ***stochastic structural stability*** could help!

Might fit in nicely with recent taste for "stochastic parameterizations"

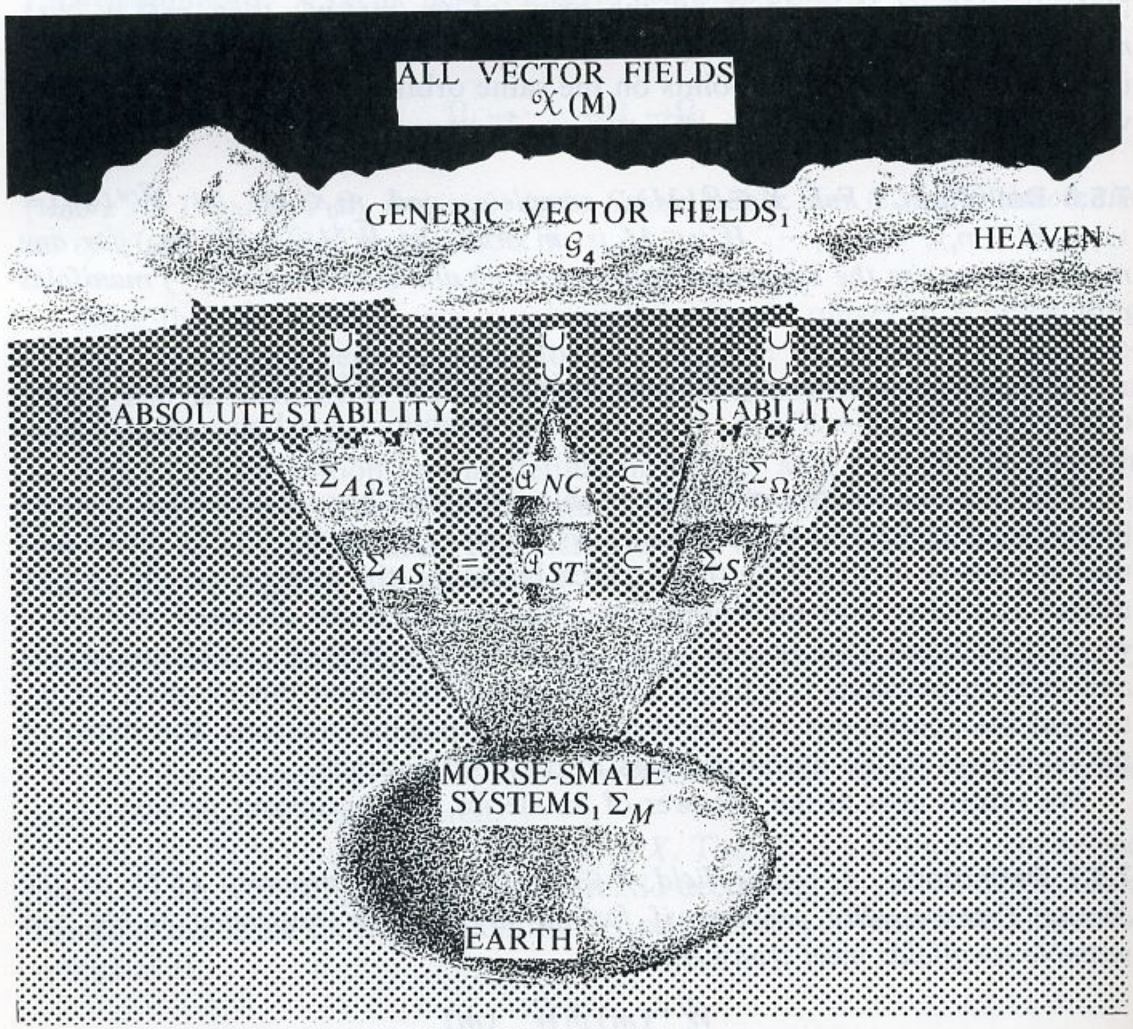


Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from *Abraham & Marsden*, 1978)

Climate and Its Sensitivity

Let's say CO_2 doubles:

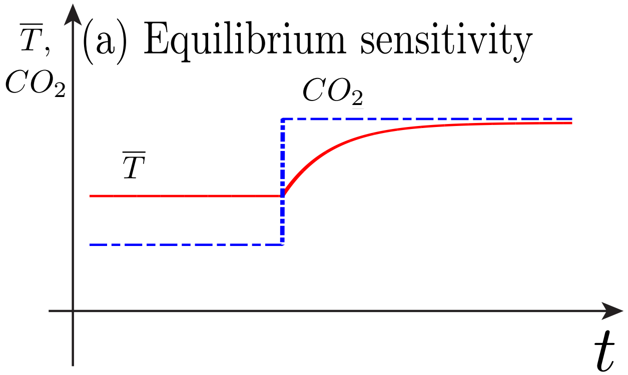
How will "climate" change?

1. Climate is in stable equilibrium (fixed point); if so, mean temperature will just shift gradually to its new equilibrium value.

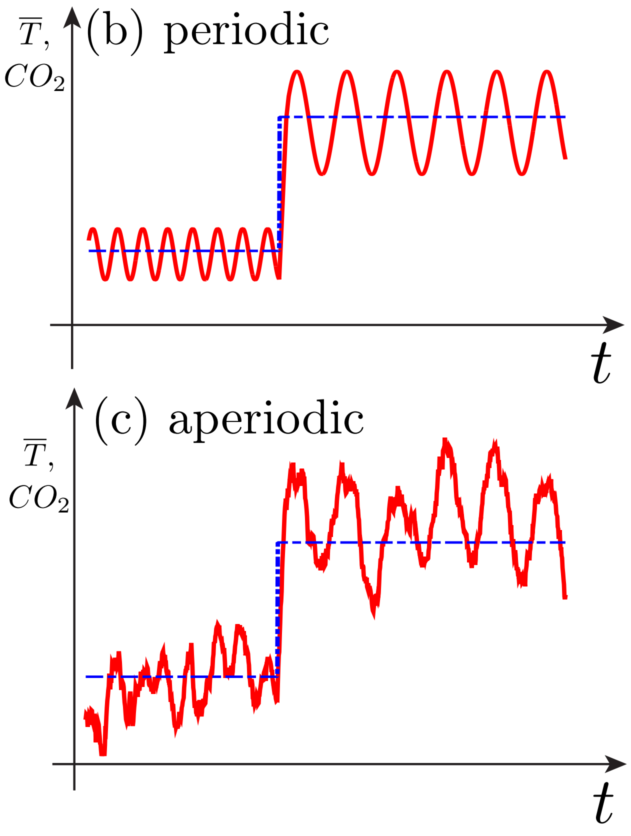
2. Climate is purely periodic; if so, mean temperature will (maybe) shift gradually to its new equilibrium value. But how will the period, amplitude and phase of the limit cycle change?

3. And how about some "real stuff" now: chaotic + random?

Ghil (in *Encycl. Global Environmental Change*, 2002)



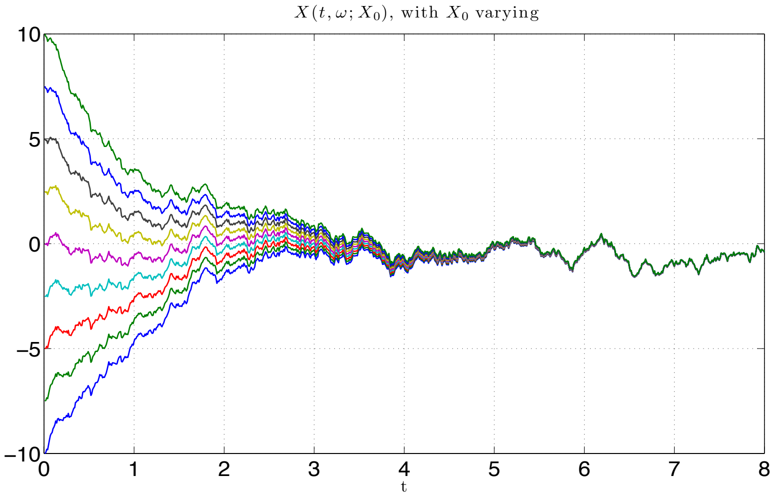
(b, c) Non-equilibrium sensitivity



A little history of climate & stochasticity

- A. Einstein’s (1905) Brownian motion paper.
- K. Itō (prof. at Kyoto U., RIMS director) formulates Itō calculus in 1942, enables solution of stochastic differential equations (SDEs); Itō’s lemma is the stochastic counterpart of Leibniz’s chain rule for differentiation.
- K. Hasselmann (*Tellus*, 1976) describes climate as Brownian motion, with weather the stochastic driver.
- In this view, the deterministic part of the model is stable, and random perturbations decay to the mean.

Kiyoshi Itō



Auto-regressive (AR) decay

Time-Dependent Forcing – I

- So far, we have dealt with governing equations that did not have explicit time dependence in the forcing or coefficients.
- This helped us understand the difficulties with weather prediction, but is clearly not good enough for studying human, and other, effects on climate evolution.
- How can we take into account both natural and anthropogenic forcing?
- As the energy being put into the system by the forcing is dissipated to smaller and smaller scales, we expect things to change in time. How they do change will depend on how the system behaves in the absence of forcing: steady state, periodic, chaotic?
- Let’s see!

Time-Dependent Forcing – II

Consider the scalar, linear ordinary differential equation (ODE)

$$\dot{x} = -\alpha x + \sigma t\,, \quad \alpha > 0\,,\ \sigma > 0\,.$$

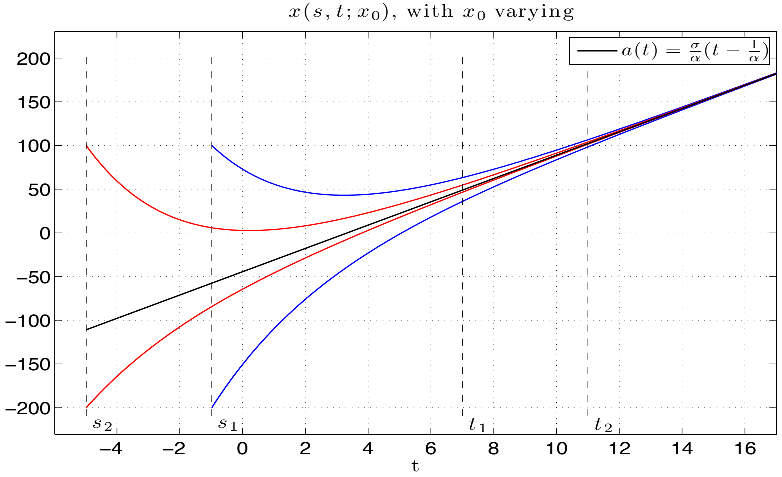
When there's no forcing, $\sigma = 0$, the ODE is purely dissipative

$$\dot{x} = -\alpha x\,,$$

and all solutions converge to the fixed point $x = 0$ as $t \to +\infty\,.$

Now what about when we do have forcing, $\sigma \neq 0$?
At each time $t = t_1$, say, we have to "pull back" and start at some time $s = s_1 << t_1$, say, to see where the flow takes us at $t = t_1$.
As $s \to -\infty\,,$ we get the pullback attractor $a = a(t)$ in the figure,

$$a(t) = \frac{\sigma}{\alpha}\left(t - \frac{1}{\alpha}\right).$$



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  - sensitivity to model formulation ➔ see below!
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  - non-autonomous and random dynamical systems (NDDS & RDS)
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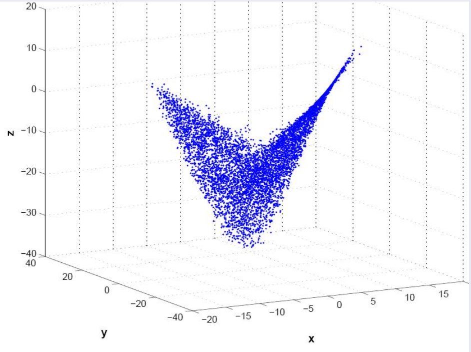
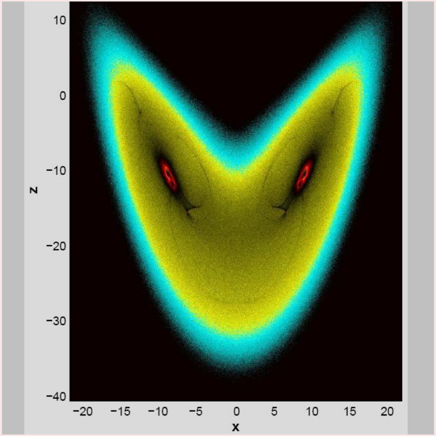
Random attractor of the stochastic Lorenz system

Snapshot of the random attractor (RA)



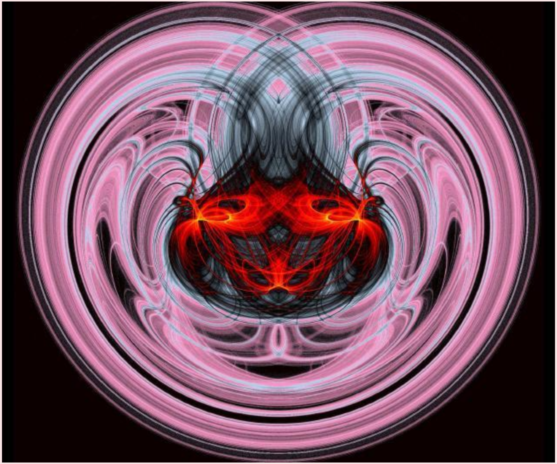
- A snapshot of the RA, $\mathcal{A}(\omega)$, computed at a fixed time t and for the same realization ω; it is made up of points transported by the stochastic flow, from the remote past $t - T$, $T >> 1$.
- We use multiplicative noise in the deterministic Lorenz model, with the classical parameter values $b = 8/3$, $\sigma = 10$, and $r = 28$.
- Even computed pathwise, this object supports meaningful statistics.

Sample measures supported by the R.A.



- We compute the probability measure on the R.A. at some fixed time t, and for a fixed realization ω. We show a "projection", $\int \mu_\omega(x, y, z)\mathrm{d}y$, with multiplicative noise: $\mathrm{d}x_i$=Lorenz$(x_1, x_2, x_3)\mathrm{d}t + \alpha\ x_i \mathrm{d}W_t; i \in \{1, 2, 3\}$.
- 10 million of initial points have been used for this picture!

Sample measure supported by the R.A.



- Still 1 Billion I.D., and $\alpha = 0.5$. Another one?

Sample measure supported by the R.A.

Sample measures evolve with time.

- Recall that these sample measures are the frozen statistics at a time *t* for a realization ω.
- How do these frozen statistics evolve with time?
- Action!

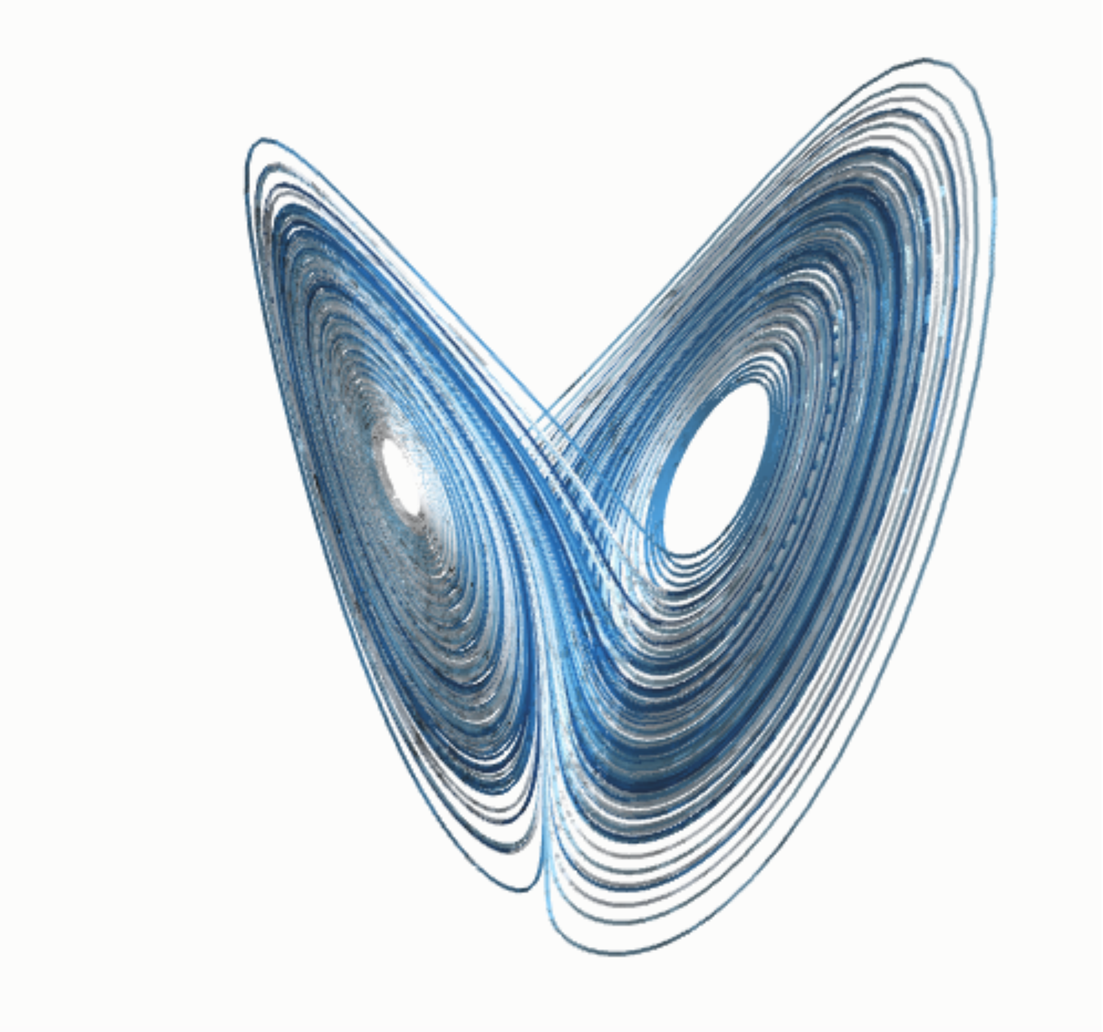
A day in the life of the Lorenz (1963) model's random attractor, or LORA for short;
see SI in Chekroun, Simonnet & Ghil (2011, *Physica D*)

Classical Strange Attractor

Physically closed system, modeled mathematically as autonomous system: neither deterministic (anthropogenic) nor random (natural) forcing.

The attractor is strange, but still fixed in time ~ “irrational” number.

Climate sensitivity ~ change in the average value (first moment) of the coordinates (x, y, z) as a parameter λ changes.



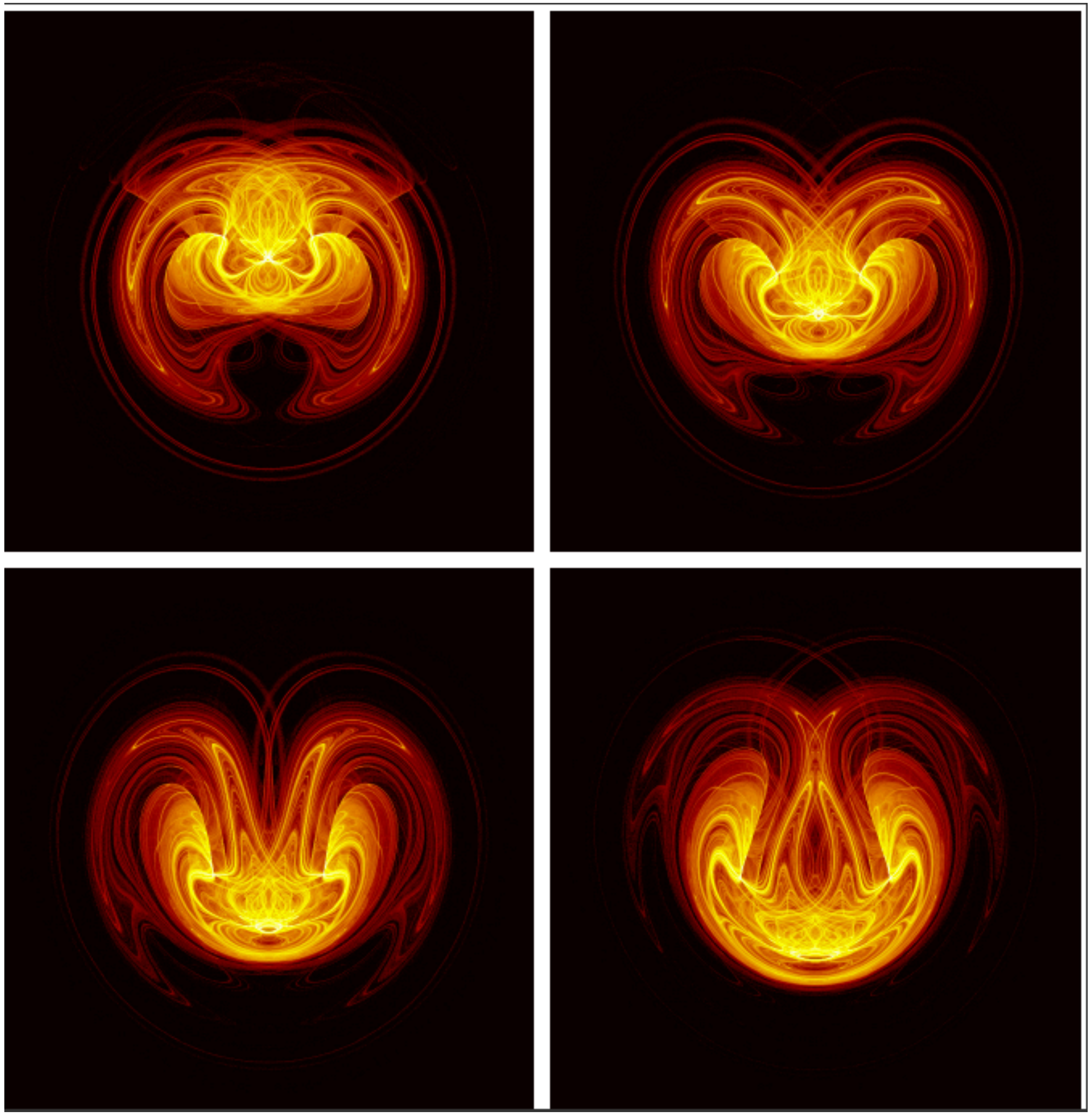
Random Attractor

Physically open system, modeled mathematically as non-autonomous system: allows for deterministic (anthropogenic) as well as random (natural) forcing.

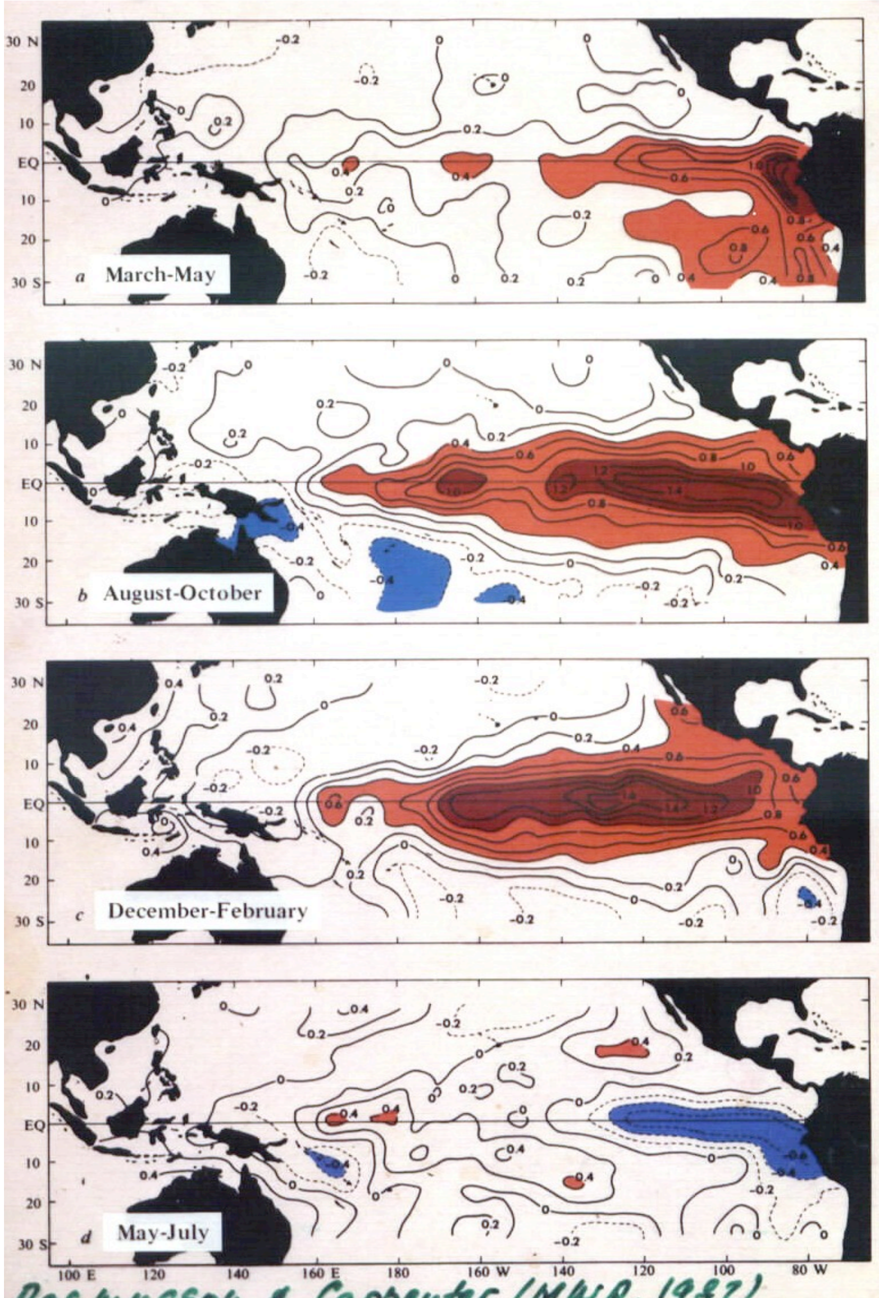
The attractor is “pullback” and evolves in time ~ “imaginary” or “complex” number.

Climate sensitivity ~ change in the statistical properties (first and higher-order moments) of the attractor as one or more parameters (λ, μ, …) change.

Ghil (*Encyclopedia of Atmospheric Sciences*, 2nd ed., 2012)



Spatio-temporal evolution of ENSO episode



Rasmusson & Carpenter (MWR, 1982)

Outline

- Weather & chaos
  - a little history of deterministic chaos
  - the Lorenz convection model & the butterfly effect
  - sensitivity to initial data ➔ local error growth
- Climate & randomness
  - the IPCC process: results and uncertainties
  - sensitivity to model formulation ➔ errors in the statistics
- Uncertainties and how to fix them
  - structural stability and other kinds of robustness
  - time-dependent forcing & what to do with it
- Two illustrative examples
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  - selected bibliography

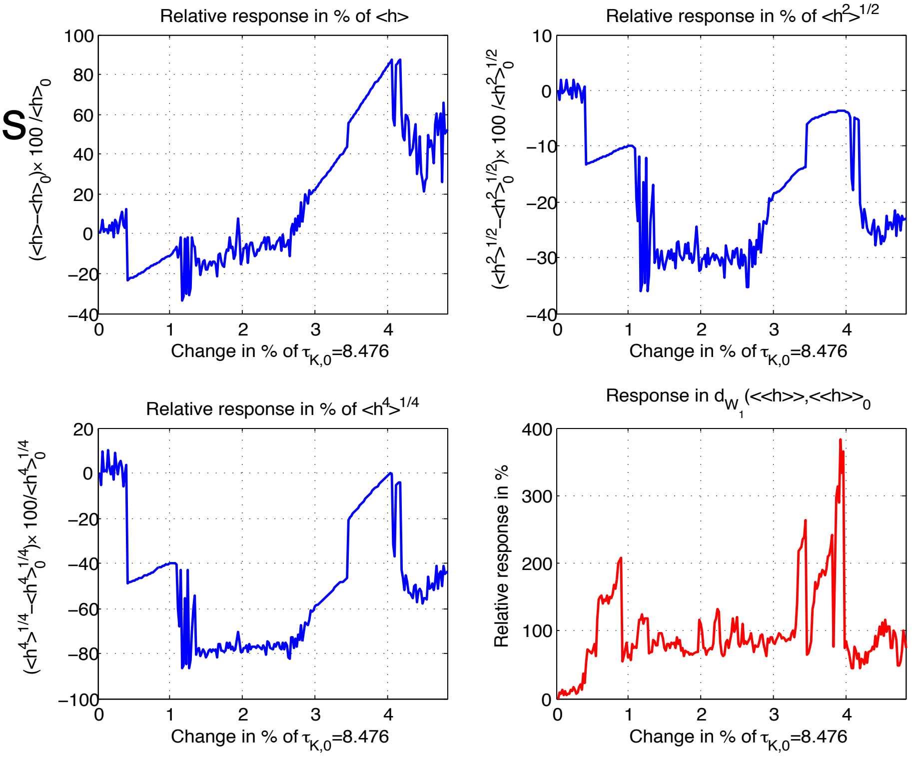
How to define climate sensitivity or, What happens when there’s natural variability?

The Galanti-Tziperman (GT) model (*J. Atmos. Sci.*, 1999)

- T is East-basin SST and h is thermocline depth;
- seasonal forcing appears explicitly;
- the model has chaotic behavior.

The figure shows the changes in the mean, as well as in two higher moments (2nd & 4th moment) of $h(t)$, along with the Wasserstein distance(*) d_W, for changes of 0–5% in a model parameter. .

Note intervals of both smooth & rough dependence!



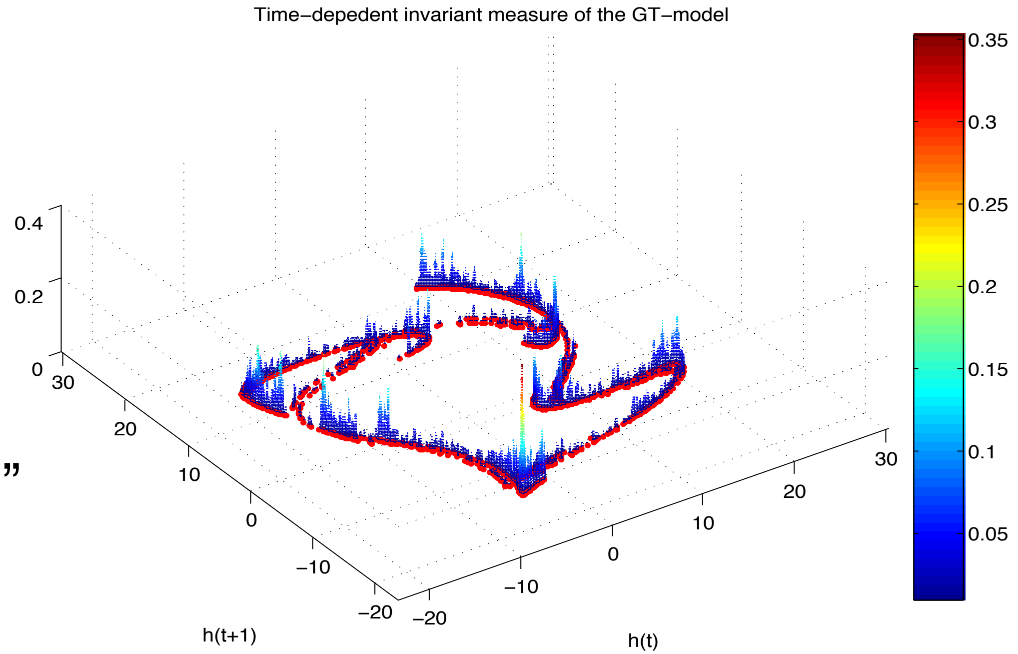
(*) Wasserstein distance d_W = “earth mover’s distance.”

Courtesy of M.D. Chekroun, work in progress

How to define climate sensitivity or, What happens when there's natural variability?

One usually defines climate sensitivity γ as Δ*T*/Δ*Q*,
where *T* is global mean temperature in ^{0}C,
and Δ*Q* is insolation change in %.
Thus γ ≈ 1 ^{0}C per 1 % change in *Q*.

But there is much more to climate than mean *T* : there is the actual distribution of temperatures in time and space, there's extrema of temperatures and of precipitations, etc., as in the figure.



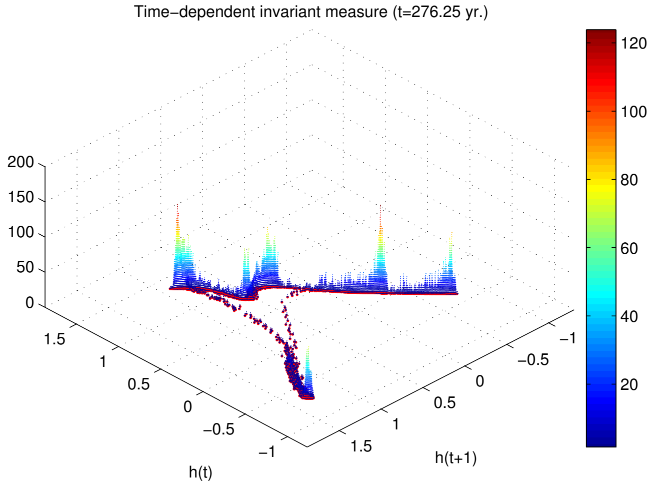
So we would like a better, more flexible definition, which does take into account these "details," as well as chaotic behavior:

$$\gamma = \partial d_{\mathrm{W}} / \partial \tau$$

How to define climate sensitivity or, What happens when there’s natural variability?

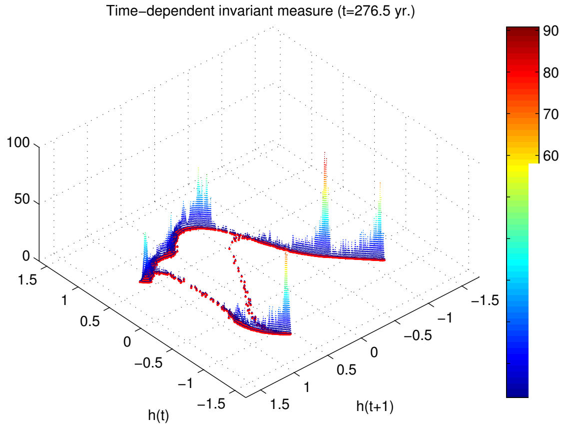
This definition allows us to watch how “the earth moves,” as it is affected by both natural and anthropogenic forcing, in the presence of natural variability, which includes both chaotic & random behavior:

:

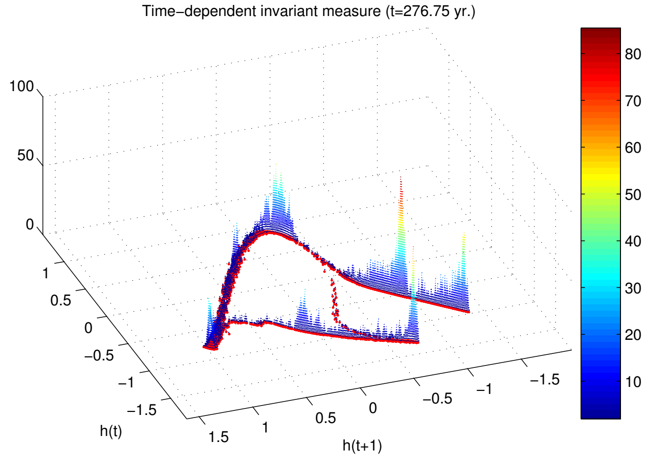


Time−dependent invariant measure (t=276.25 yr.)

$$\gamma = \partial d_{\mathrm{W}} / \partial \tau$$



Time−dependent invariant measure (t=276.5 yr.)



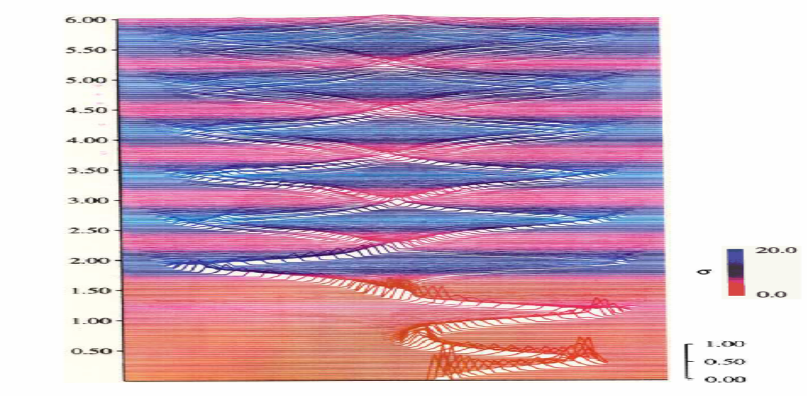
Time−dependent invariant measure (t=276.75 yr.)

Yet another (grand?) unification

Lorenz (*JAS*, 1963)

Climate is deterministic and autonomous,
but highly nonlinear.

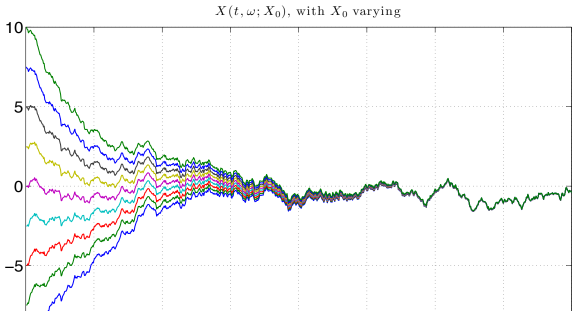
Trajectories diverge exponentially,
forward asymptotic PDF is multimodal.



Hasselmann (*Tellus*, 1976)

Climate is stochastic and noise-driven,
but quite linear.

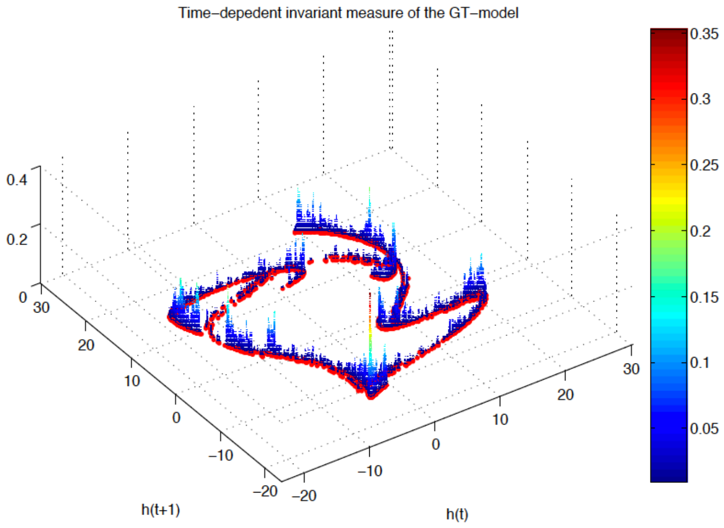
Trajectories decay back to the mean,
forward asymptotic PDF is unimodal.



Grand unification (?)

**Climate is deterministic + stochastic,
as well as highly nonlinear.**

**Internal variability and forcing interact
strongly, change and sensitivity
refer to both mean and higher moments.**



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Concluding remarks

Climate change & climate sensitivity

What do we know?

- It's getting warmer.
- We do contribute to it.
- So we should act as best we know and can!

What do we know less well?

- By how much:
  - Is it getting warmer?
  - Do we contribute to it?
- How does the climate system (atmosphere, ocean, ice, etc.) really work?
- How does natural variability interact with anthropogenic forcing?

What to do?

- Better understand the system and its forcings.
- Explore the models', and the system's, robustness and sensitivity
- Act sensibly on the available information, with wisdom & compassion!!

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Some general references

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Solomon, S., *et al*. (Eds.). *Climate Change 2007: The Physical Science Basis*. *Contribution of Working Group I to the Fourth Assessment Report of the IPCC*, Cambridge Univ. Press, 2007.

Nature is not deterministic or stochastic:

It depends on what we can, need & want to know
— more or less detail, with greater or lesser accuracy —
larger scales more accurately,
smaller scales less so

But we need both, deterministic and stochastic descriptions.
Knowing how to combine them is necessary, as well as FUN!

Reserve slides

Concluding remarks

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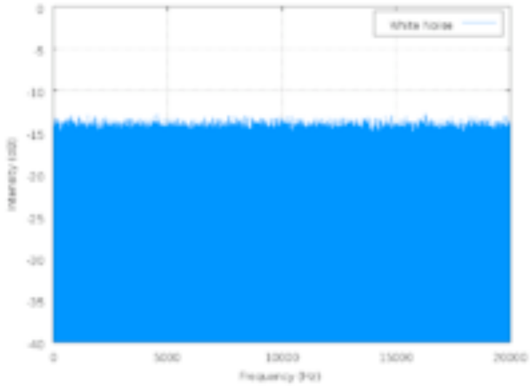
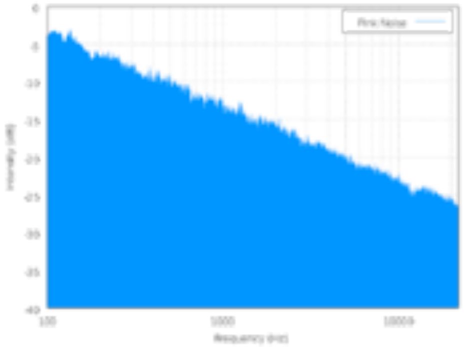
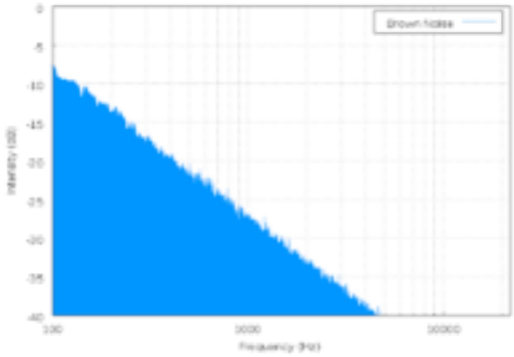
Noise “colors”



White noise, $S\sim f^0$



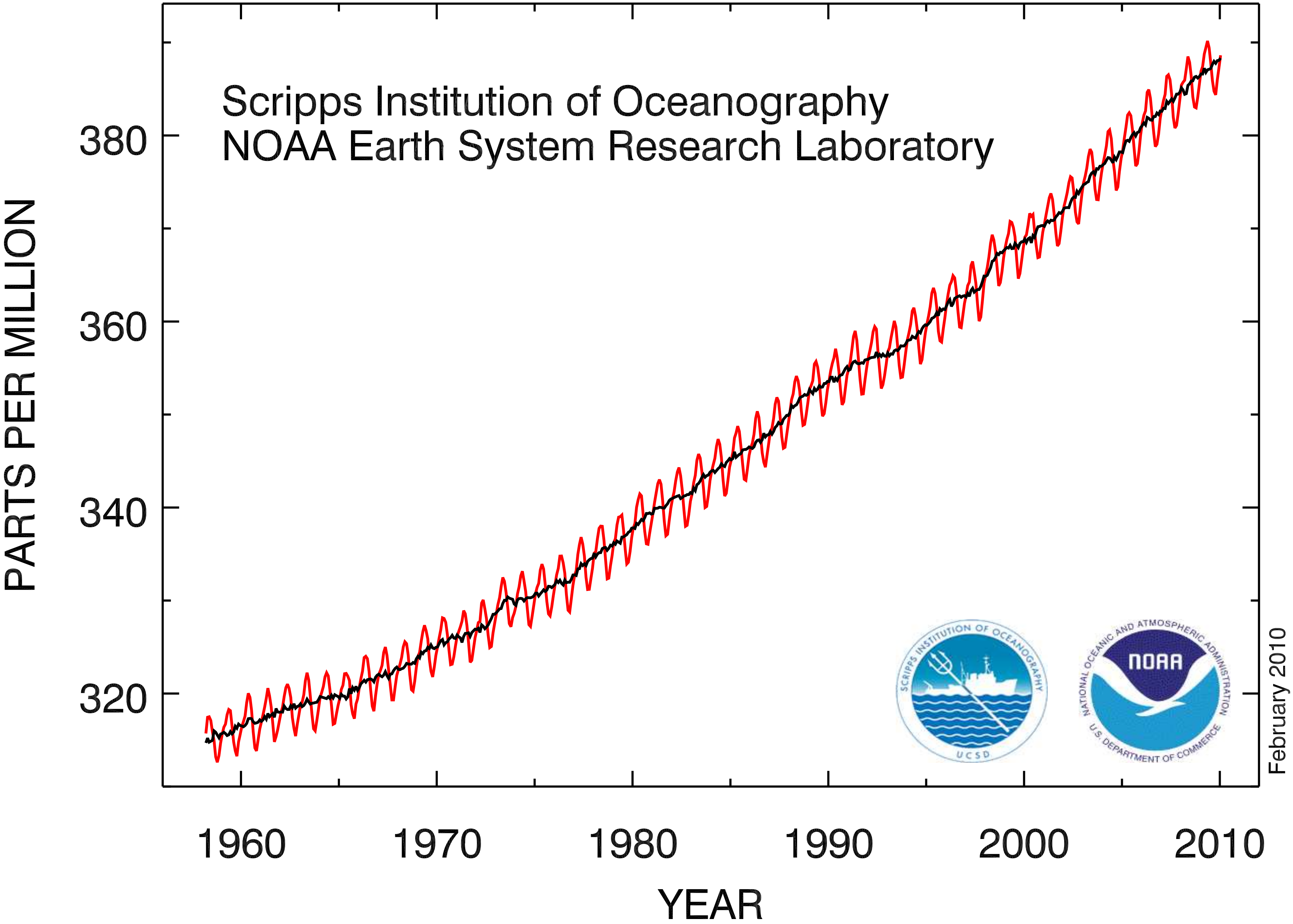
Pink (or 1/*f*) noise, $S\sim f^{-1}$



Red (or Brown) noise, $S\sim f^{-2}$

Blue noise, $S\sim f^{+1}$

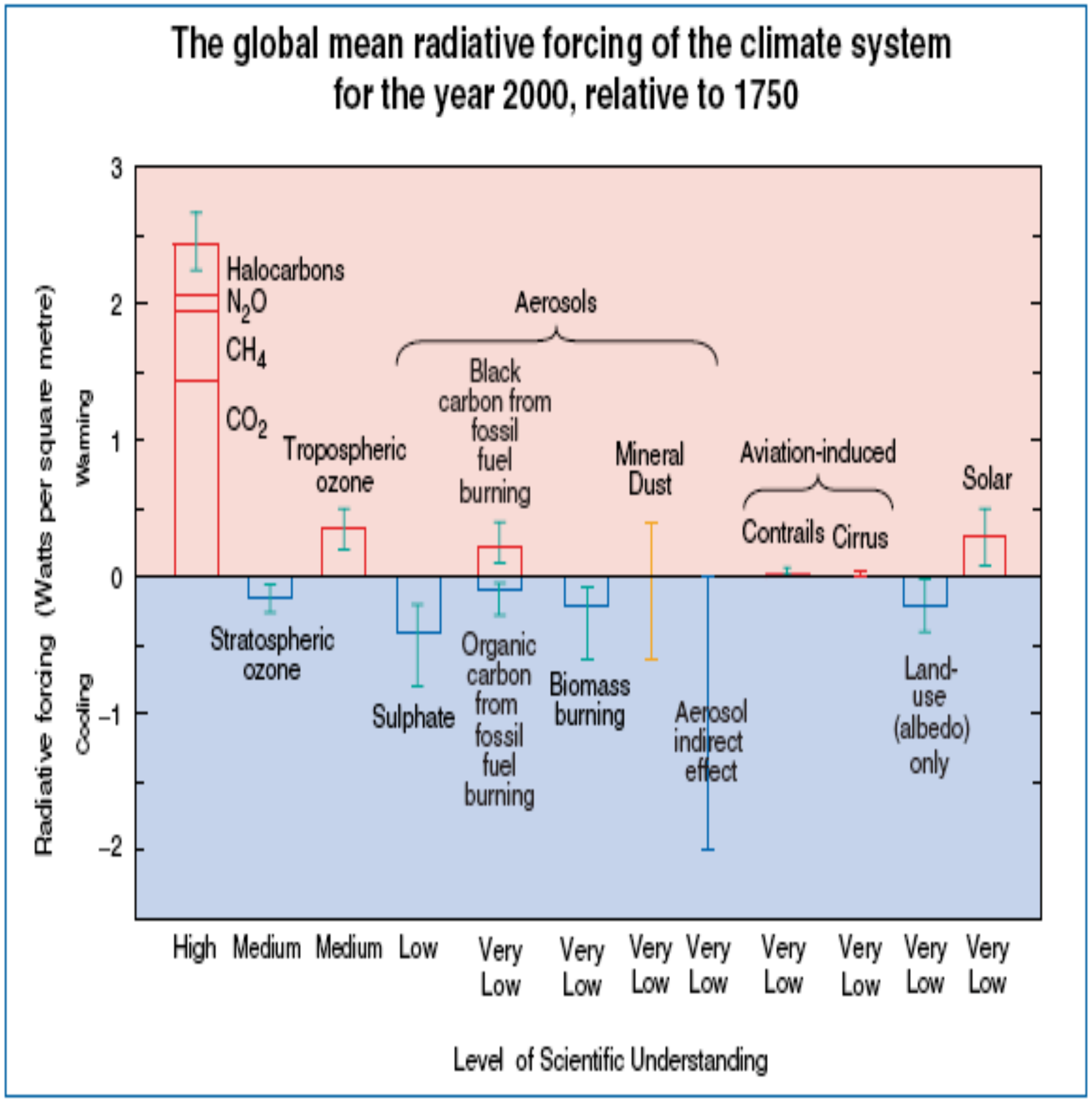
Atmospheric CO_2 at Mauna Loa Observatory



February 2010

Uncertainties in the forcing

Contributions to the forcing, natural and ***anthropogenic***, also have substantial uncertainties



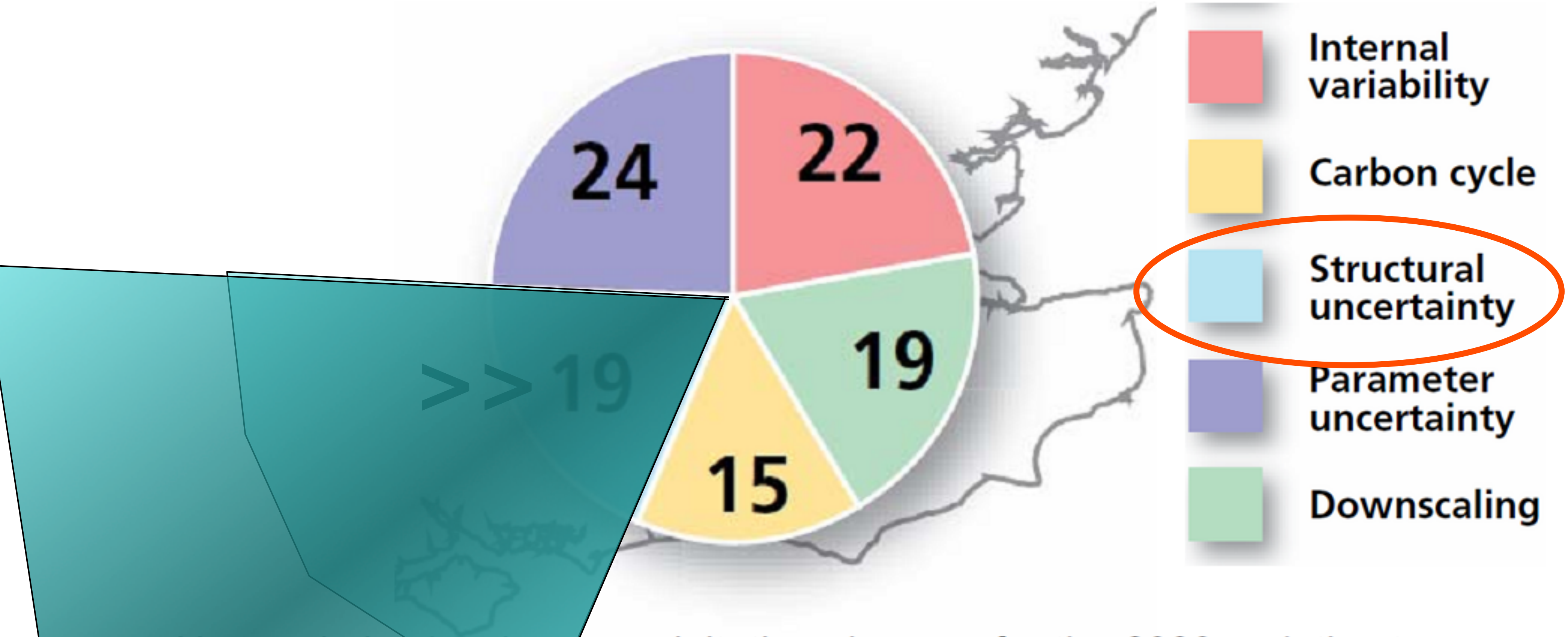
The global mean radiative forcing of the climate system for the year 2000, relative to 1750

Figure 3: Many external factors force climate change.

Source : IPCC (2001), *TAR, WGI, SPM*

How important are different sources of uncertainty?

- Varies, but typically no single source dominates.

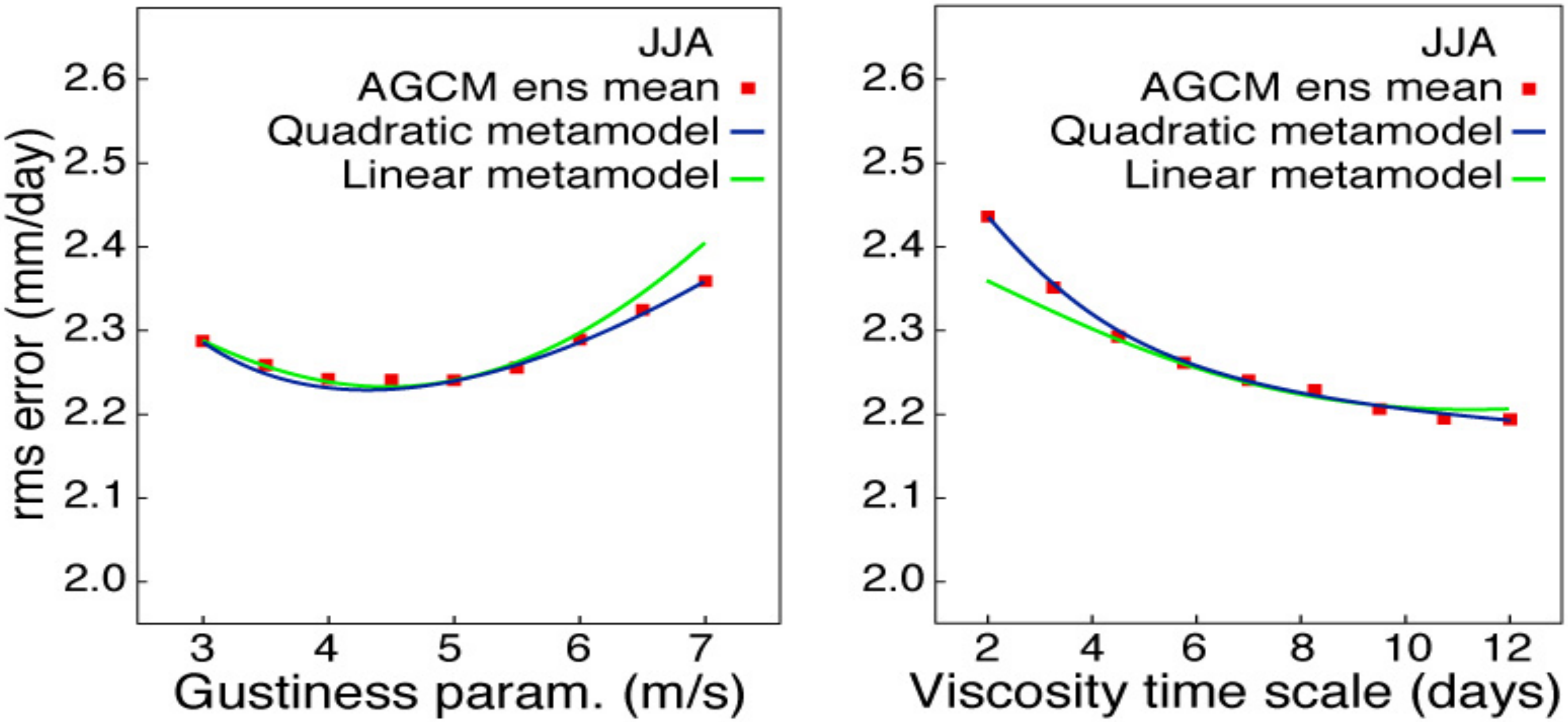


Uncertainties in winter precipitation changes for the 2080s relative to 1961-90, at a 25km box in SE England

Source: Met Office

Parameter dependence – II

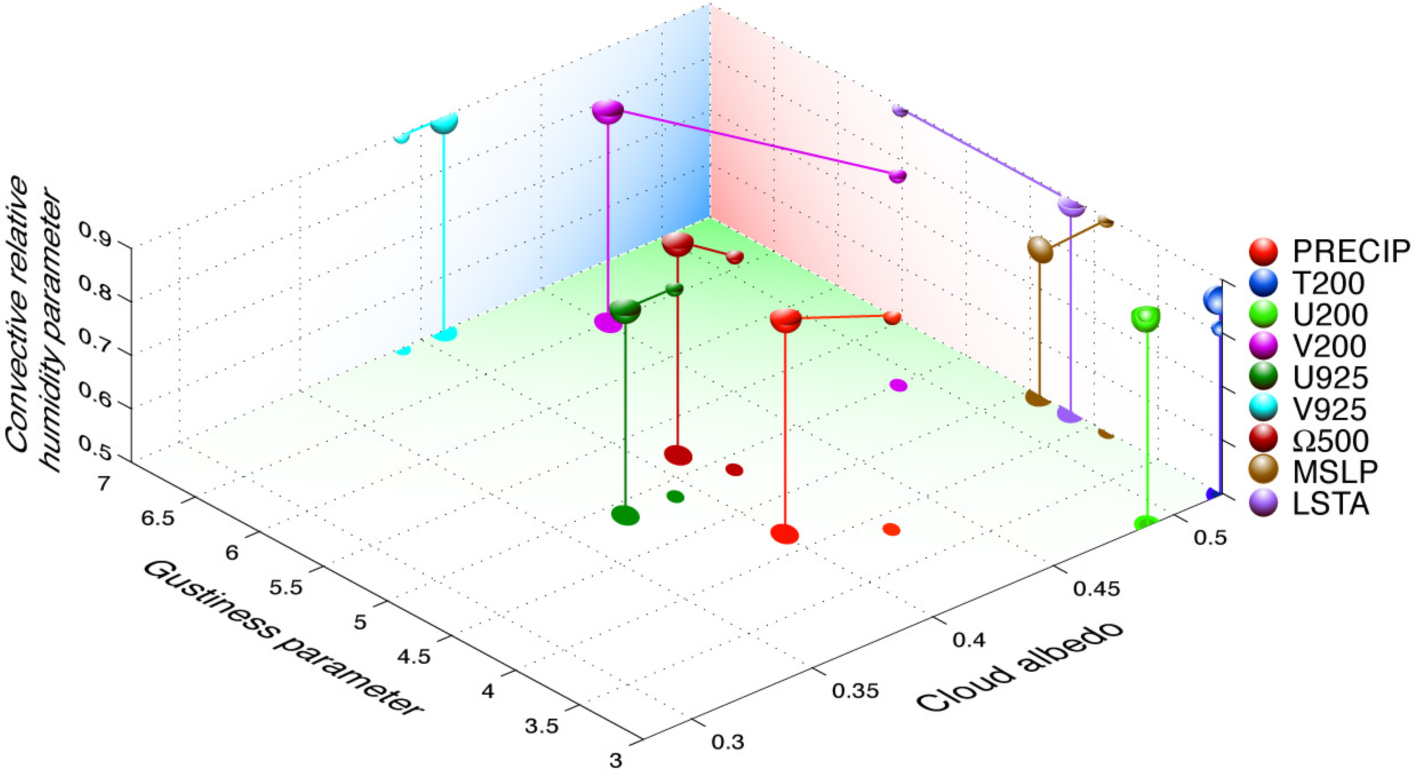
When it is smooth, one can optimize a GCM's single-parameter dependence



ICTP AGCM (Neelin, Bracco, Luo, McWilliams & Meyerson, *PNAS*, 2010)

Parameter dependence – III

Multi-objective algorithms avoid arbitrary weighting of criteria in a unique cost function:



Optimization algorithms that are $\mathcal{O}(N)$ and $\mathcal{O}(N^2)$, rather than $\mathcal{O}(S^N)$, where N is the number of parameters and S is the sampling density.

ICTP AGCM (Neelin, Bracco, Luo, McWilliams & Meyerson, *PNAS*, 2010)

Climatic uncertainties & moral dilemmas

Thought leaders *Rice, top left, spoke of multilateralism, while Bono, left, demanded more action on poverty. Presidents Karzai and Musharraf, right, both face troubles at home*

♥ *Feed the world today*

or…

♥ … *keep today's climate for tomorrow?*

Agitator *Gore wants a global compact to tackle climate change and poverty*

Davos, Feb. 2008, photos by *TIME Magazine*, 11 Feb. '08;

see also Hillerbrand & Ghil, *Physica D*, 2008, **237**, 2132–2138, doi:10.1016/j.physd.2008.02.015 .

The Biofuel Myth

- Fine illustration of the moral dilemmas (*).
- Conclusion: “**festina lentae**,” as the Romans (**) used to say..

(**) ~ Han dynasty

(*) Hillerbrand & Ghil, *Physica D*, 2008, doi:10.1016/j.physd.2008.02.015, available on line.

APRIL 14, 2008

A Baghdad Diary: Why Iraq Hasn’t Healed

Italy’s Pols Should Look To the South

TIME’s 85th Anniversary: A Timeline

TIME

The Clean Energy Myth

BY MICHAEL GRUNWALD

Politicians and Big Business are pushing biofuels like corn-based ethanol as alternatives to oil. All they’re really doing is driving up world food prices, helping to destroy the Amazon jungle, and making global warming worse

www.time.com

Climate Change 1816–2008

T. Géricault, 1819,
Le Louvre

M. Gillot, 2008,
Le Monde